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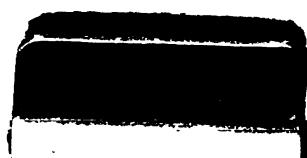
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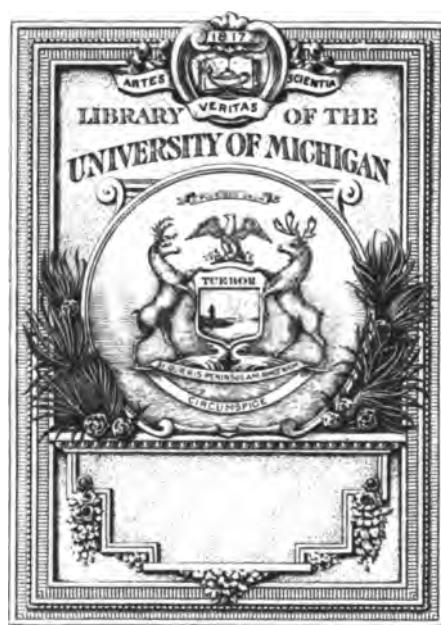
QA
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1654

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The Gift of
WILLIAM H. BUTTS, Ph.D.
A.B. 1878 A.M. 1879
Teacher of Mathematics
1898 to 1922
Assistant Dean, College of Engineering
1908 to 1922
Professor Emeritus
1922



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16.4





PHILIPPE^{van} LANSBERGH

TRIANGULORUM
GEOMETRIÆ
LIBRI QUATUOR;

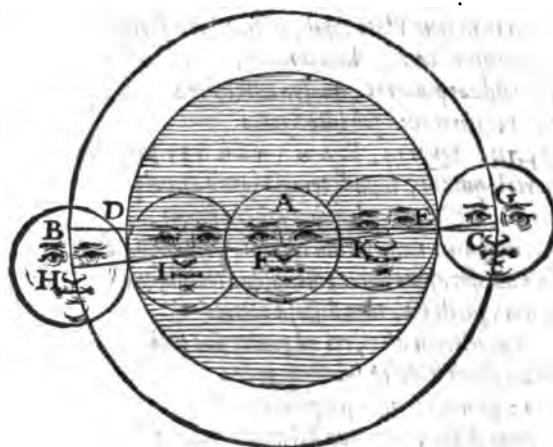
In quibus novâ & perspicuâ methodo, & ~~admodum~~ tota ipsorum
Triangulorum doctrina explicatur.

Item

PHILIPPI LANSBERGII

CYCLOMETRIÆ NOVÆ
LIBRI DUO.

Ab Autore recognita, multisque in locis aucta.



MIDDELBURGI ZELANDIA,

Apud Zachariam Roman, Bibliopolam.

C I S I O C I I I I I.

IN
PHILIPPILANSBERGII
TRIANGULORUM
QA G E O M E T R I A M
33 L29 I A N I D O V S & F I L I I C A R M E N .
1654

Felix ille animi nimis , egregiusque laboris ,
Quem juvat aſſiduè nisi praſtantibus aufis ,
Poſit ut infectas terras & ſidera mente ſequentem
Et penitus patrio mentem defigere caelo .
Namque illum aeterni Patrio indulgentia major
Linquentem terras & ſidera mente ſequentem
Excipietque polo , & fulgentibus inferet aſtris .
Crediderim haud aliter priſcos agitare parentes ,
Qui primi aſtrorum leges atque aetheris omnes
Rechafere vias ; & Mundi ſtammea retta
Accessere acie mentis , doctamque per artem
Orbibus affixere ſuis palantia ſigna .

Abrahamus . Qualis & ille * Senex , ſtructa cui filius arā
Mactandus ſodis ; & Setbi antiquior illo
Progenies dupli cielum ſcrutata columnā .
Nec non Cauſaſē pendens de rupe Prometheus ,
Qui tenuem nividis ignem furatus ab aſtris
Fixerat humanos glebaque & fulmine vulnus .
Et tu , quem Oetea rapiuere ad ſidera flamma
Atque tuo quondam libratum vertice cielum .

De Hercule Astro-
logo vide Festum .
Berosus . Tum * cui conſpicuam erexit ſtatuum Attica tellus ,
Et voluit fulvo lingua fulgere metallo .

Vi faceam te magne Plato , qui , ut in aethora ferret
Sublimes oculos , homini data lumina , dixi .
Ac tot Chaldaeos proceres , quoſye extulit ora
Aſſiria , vel ubi mediā ſub luce Syenes

Vimbra pegit . Quos tu , L A N S B E R G I , pone ſecutus
Nil mortale putans , liquidi templa ignea Mundi
Percurrens , ſtellisque ardentiibus aethera fixum ,
Tam certus ſtatiuſ numerorum includis Olympum , &
Monima mēſurā ſque doces , flexuſque recessuſque
Innumeratos ; facili tot , tam diuſa coerces
Gyro ; ut prodiuſem aſtriferi ad penetralia cieli

id eſt Arithmetice
& Geometricæ . Ostendas callem * docta ſubmixta arena
Remigio ; ſic non humeris ſed pectore cielum
Fulciit altus Atlas ; nec equo ſed mente volavit

Bellerophon . Atque animi permis liquidi ad confinia Mundi
Ille , Chimeraeas potuit qui vincere flamas .
Nobilis & ſummo nunc ſplendet in aethere Perſeus
Gorgonis anguicomma domitor , qui nubila ſupra
Ventorumque leves animas & fulmina vextus
Ingenio acceſſit ſuperum tonitralia templa .

Ad

EES

I. H. Butt
Whelton
14-32
33358

Ad Amplissimos & Magnificos Viros,

C O N S U L E S,

Totumque inclytae Middelburgensium Reipub.

S E N A T U M,

Dominos suos plurimum observandos.

P H I L I P P V S L A N S B E R G I V S .



DEFERO vobis Amplissimi Viri, libros, quos de Triangulorum Geometria quos primū in urbe vestra concepi, post Goesæ scripsi, & perscripsi; nunc verò, quantā à me fide potuit, & diligentia recognovi. Sed quod scriptores ferè omnes in operum suorum p̄fationibus facere consueverunt; ut & lucubrationum suarum rationem, & dedicationis causas exponant; id mihi potissimum faciendum duxi. Iam nunc enim mihi illos audire videor qui me & imprudentiæ, & temeritatis accusent. Imprudentiæ quidem, quod eam Geometriæ partem explicandam suscepimus, in cuius demonstratione feliciter laborarunt non pauci ex priscis Mathematicis; & quam nostro etiam seculo multi magnique viri scriptis suis illustrarunt: temeritatis verò, quod primum hunc, rudemque ingenii mei fœtum, Amplitudini vestræ offerre ausus sim. Sed facilis erit utriusque criminis dilutio, apud eos, qui rem ipsam aestimare, & cæcos animi affectus (interea dum ipsis respondero) deponere voluerint.

Quod ad primum, hoc sanctè affirmo, non eo animo laborem hunc nobis suscepimus esse, ut eorum monumenta qui ante nos scripsierunt, & immortalitati consecrata sunt, aut improbemus, aut è manibus studiosorum abducamus: Veneramur enim, & suspicimus omnes, qui in hoc scribendi genere versati sunt; imò aliorum scriptis non mediocriter adjutos esse ingenuè fatemur: Ingenii enim est (ut inquit ille) fateri à quo profeceris. Sed quia plærique ita scripsere, ut doctioribus tantum scripsisse videantur; &

fusius quam ut exiguo tempore perlegi possint : non inanem operam positurum me putavi ; si rudiores instituendos deligerem , & compendio doctiores juvarem . Feci igitur quantum potui ; nihil ad ostentationem , nihil invidiae causâ : hic unus mihi scopus propositus ut multis prodessem . Quod spero me assequutum esse : et si enim de utilitate operis , laborisque mei , alios ; non me verba facere oporteat : hujus tamen plus se hinc cepisse fatebitur Lector Philomathes (si animum intenderit) quam ego verbis verecundè spondere ausim .

Cæterum quod in nomine vestro , hanc meam lucubrationem lucem ad spicere voluerim ; nemo temeritati tribuat : nam ut hoc facerem , multæ mihi gravesque causæ fuerunt . Prima , quod illiberabilis & ingratiani Æimi esse judicabam , hoc mei ingenii foetu in vestrâ urbe primùm formato , Amplitud . V. tanquam seminis vestri proventu malignè spoliare . Altera , quod si labor hic noster literariæ Réipublicæ utilis futurus esset , sub vestro nomine longè gratissimum futurum putabam . Nam ut Amplitud . V. gloriosum est ; ita studiis cum primis utile , tantos viros Mathematicarum artium patronos esse . Postremo vestra illa in bonos humanitas , & maximè eos , quos aliquod doctrinæ nomen commendat aut literarum , impulit me , ut vicissim ego hoc gratitudinis officio , meam erga vos voluntatem fidemque testarer . Quapropter cùm tot tantæque mihi causæ munusculi mei Amplitud . vestræ offerendi fuerint ; ut pro vestra summa æquitate benignè accipiatis , & certissimum meæ erga vos universos & singulos observantiaæ ~~universos~~ esse statuatis , obnixè rogo : Ita enim laboris operæque meæ uberrimum fructum percepisse videbor . Valete Amplissimi & Magnifici Viri . Goesæ , 111 Kalend . April . Anno Christi c l o I o x c i .

G E O M E T R I A E
T R I A N G U L O R U M
L I B E R I.

De magnitudine rectarum linearum que circa Circuli peripheriam considerantur.

I.

T R I A N G U L O R U M Geometria est, quæ ex tribus quibuscumque, vel angulis, vel lateribus, in rectilineo aut Sphærico Triangulo datis, reliquorum laterum angulorumque dimetendorum rationem tradit, adminiculo Canonis Triangulorum, ex magnitudine restarum linearum, quæ circa circuli peripheriam considerantur, compositi.

Suscepta nobis est explicanda Triangulorum Geometria, rectè igitur à definitione ejus auspicamur: omnis enim quæ à ratione suscipitur de aliqua re institutio, debet à definitione profici, ut intelligatur id de quo disputatur. Definitio autem premissa cum à partibus totius doctrine sumpta sit, validè clara est, & sigillatim deinceps demonstrabitur.

N O T I Z M A .

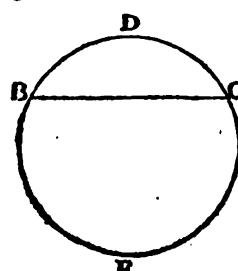
Eius itaque partes tres sunt. Prima ex primis Geometriæ elementis, rectarum linearum magnitudinem, quæ circa circuli peripheriam considerantur, demonstrat: Altera Canonis Triangulorum ratiō: Postrema, usq; ejus in calculo Triangulorum rectilineorum, & Sphæricorum.

2. Rectarum vero linearum quæ circa circuli peripheriam considerantur, aliae sunt in circuli peripheria, aliae extra, aliae per circuli peripheriam.

Veteres Mathematici cùm solis subtensis in Triangulorum Geometria intererant, rectarum solammodo magnitudinem quæ in circulo sunt investigabant. Nobis vero cùm plenior, planiorque mensurandi ratio explicanda sit, etiam earum quæ extra & per circuli peripheriam sunt, magnitudo demonstranda est.

De magnitudine rectarum in Circuli peripheria.

3. In circuli peripheria considerantur Subtensa, & Sinus.
4. Subtensa est recta linea in circulo, dirimens eum in duo segmenta; & utrumque pariter subtendens.



Talis est in adjecto schemate recta B C. dirimit enim circulum B D C E in duo segmenta, B D C & B E C: & utrumque pariter subtendit.

5. Sinus, est recta linea in semicirculo, ab arcus termino perpendicularis.

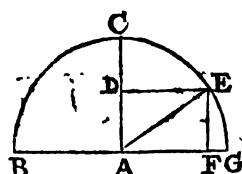
Vox Sinus Arabica est, & proinde barbaræ; sed cum longo usu approbaræ sit, & commodior non suppetat, nequaquam repudianda est: faciles enim in verbis nos esse oportet, cùm de rebus convenit.

6. Sinus rectus est aut versus.

Recentiores aliqui Sinum dividunt in primum & secundum: nam cum ex premissa Sinus definitione, versus non minus perpendicularis sit quam rectus, etiam rectum esse contendunt, & proinde vitiostam distributionem Philesofbo ubi partes convenienter. Verum cum hoc verso Sinus proprium sit, quod recto versus sit, rectus solammodo nata rati; nulla causa est ab auctoritate divisione recedendi.

7. Sinus rectus est recta linea in semicirculo, ab arcus termino perpendicularis in diametrum, dividens semicirculum in duo segmenta; ad quorum utrumque pariter referuntur.

Geometriæ Triangulorum Liber I.



Talis est in adjunctâ figurâ rectâ E F; est enim ab E arcus termino, perpendicularis in diametrum BAFG; dividitque semicirculum BCEG in duo segmenta, GE, & BCE, ad quorum utrumque pariter referatur.

Π Ο Ρ Ι Σ Μ Α.

Itaque Sinus rectus, est semissis Subtensæ arcus dupli.

Nam quod Subtensa est in circulo, id Sinus rectus est in semicirculo, quemadmodum definitiones Sinus recti & Subtensa, inter se collata ostendunt.

8. Sinus rectus peripheriaz, & complementi sui æque possunt radio.

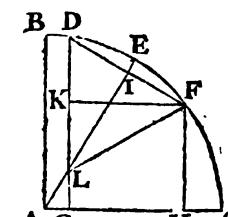
Complementum peripheria dicimus reliquam peripheriam data ad circuli quadrantem. Sit igitur in premissa figura, recta E F, sinus rectus peripherie G E vel B C E: & complementi sui C E sinus rectus E D, vel equalis illi A F per trigeminam quartam primi elementorum. Dico A F & E F, æque posse radio A E. Nam per penultimam primi Euclidis, in Triangulis rectangularibus, quadrato laterum rectum angulum continentium, equalia sunt lateri rectam angulum subiendenzi. Sed A F E est Triangulum rectangularum ad F per septimam hujus, crux vero rectum ambienzia sunt A F & E F: æque possum ergo radio A E rectum angulum subiendenzi; quod erat demonstrandum.

Π Ο Ρ Ι Σ Μ Α.

Itaque dato radio cum sinu recto peripheriaz, datur etiam sinus rectus complementi sui: dempto enim sinus noti quadrato ex quadrato radii, relinquitur quadratum sinus complementi; cuius radix est ipse sinus quadratus.

In exemplo sit radius A E 10, & E F 6; erit D E 8: ablato scilicet quadrato E F 36, ex quadrato radii A E 100, & residu 64, quadrato latere 8 assumpto.

9. Differentia Sinuum rectorum peripheriarum duarum, à circuli sextante æqualli intervalllo remotarum, æquatur Sinu recto peripheriaz alterutrius, à circuli sextante intervalli.



Sint in quadrante A B C peripherie dua C F & C D, equali intervalllo ab E circuli sextante remota; & barum recti sinus F H & D G: differentia vero sinuum D K. Dico D K differentiam sinuum rectorum peripheriarum datarum, equali D I vel F I, alterutrius peripherie à circuli sextante intervalllo! Triangulum enim D L F est equiangulum (nam D L latus Trianguli rectanguli D I L, equatur L F lateri Trianguli rectanguli F I L per quartam primi elementorum: & proinde anguli ad D & F in Triangulo D L F per quintam ejusdem equales sunt) Sed angulus D L E est partium 30, equalis scilicet angulo B A E per secundam & quintam sexti elementorum: totu itaque D L F est partium 60. Talum vero etiam est angulus ad D & F signatim per trigeminam secundam primi elementorum. Quare cum Triangulum D L F equiangulum sit; etiam aquilaterum est per quintam ejusdem: & proinde latus D F equale lateri D L; & semissis illius D E per decimam primi elementorum, aequalis semissi hujus D K: quod erat demonstrandum.

Π Ο Ρ Ι Σ Μ Α.

Quare, si duarum peripheriarum, æqualiter à circuli sextante remotarum, recti sinus dentur, etiam distantiaz peripheriaz alterutrius à circuli sextante rectus sinus innotescet; differentia enim sinuum datorum, est ipse sinus quadratus.

In exemplo esto peripheria C F partium 50, distans ab E circuli totius sextante partibus 10; & ejus rectus sinus F H 7660: peripheria vero C D, partium 70, simili intervalllo ab E remota; & sinus rectus ejus D G 9396. Differentia sinuum DK 1736, aequalis est sinu recto arcus EF vel ED, partium 10.

Quod si rectus sinus peripheriaz alterutrius, cum sinu recto distantiaz notus sit, etiam reliqua peripheriaz rectus sinus invenietur: ablato enim sinu recto peripheriaz distantiaz, ex sinu recto peripheriaz sextante circuli majoris, relinquitur sinus rectus peripheriaz minoris; adjecto vero eodem sinu distantiaz ad sinum rectum peripheriaz sextante circuli minoris, componitur sinus rectus peripheriaz majoris.

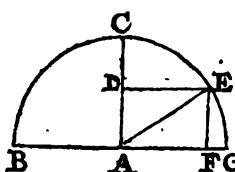
In eodem exemplo auferatur sinus rectus distantia F I, vel equalis ei D K 1736, ex D G 9396, sinu recto peripheria C D, circuli sextante majoris; relinquitur K G, vel equalis illi F H per trigeminam

Geometriæ Triangulorum Liber I.

3

Sumam quartam primi elementi. 7660, sinus rectus C F peripherie minoris. Addatur viceversa in unam summam sinus rectus D K 1736, & sinus rectus F H, vel K G, 7660; componitur sinus rectus D G 9396, competens peripheria C D, sextante circuli majori.

10. Sinus versus, est recta linea in semicirculo, ab arcus termino altero, ad sinum rectum perpendicularis.



Talis est recta G F, est enim perpendicularis ab altero termino peripherie G E, nempe G, in sinum rectum E F. Item B F: nam & ea perpendicularis est à peripherie B C E, termino altero B, ad E F sinum rectum peripherie ejusdem.

11. Sinus peripherie versus, & complementi sui rectus æquantur radio.

Sic in figura superiori, recta F G, sinus versus peripherie G E; & A F, sinus rectus complementi sui equantur radio A F G. Nam per communem sententiam, Totum equale est omnibus partibus suis simul sumptis.

Π Ο Ρ Ι Σ Μ Α.

Proinde radio dato, & sinu recto complementi peripherie, datur ipsius peripherie sinus versus. Dempto enim sinu recto complementi peripherie ex radio, relinquitur sinus versus peripherie datæ, quadrante circuli minoris: adjecto vero sinu recto excessus peripherie super circuli quadrantem ad radium, componitur sinus versus peripherie datæ; quadrante circuli majoris.

In exemplo datur radius A G 10, & A F 6, rectus sinus peripherie E C, complementi E G ad circuli quadrantem: erit F G 4, sinus versus peripherie E G, quadrante circuli minoris. Rursus, fit C E, excessus peripherie B C E, super circuli quadrantem B C; & sinus rectus ejusdem D E vel A F 6, radius A B ut supra 10: erit B A F 16, sinus versus peripherie B C E, quadrante circuli majoris.

12. Sinus rectus & versus, æque possunt sui arcus subtensa.

B. Sit in quadrante B C D E, C F sinus rectus arcus C E; E F ejusdem peripherie sinus versus: & subtensa ejusdem C H E. Dico, C F sinum rectum, & E F versum, æquari C H E, subtensa arcus sui C D E. In rectangulis enim triangulis per penultimam primi Elementorum quadrata laterum rectum ambientium, æquantur quadrato lateris recto angulo oppositi: Sed Triangulum C F E, est rectangulum ad F per septimam bujus: Latera vero rectum ambientia sunt sinus C F & E F; oppositum recto angulo latus est C E, subtensa arcus C D E. Itaque quadrata sinuum C F & E F, æquantur quadrato subtensa C E: quod erat demonstrandum.

Π Ο Ρ Ι Σ Μ Α.

Quare cujusvis peripherie recto sinu, & verso cognito, invenitur & subtensa ejus; & sinus rectus peripherie diuidiz: quadrati enim recti sinus, & versi peripherie aggregati radix, datæ peripherie subtensa est; & semissis ejus, est sinus rectus peripherie diuidiz.

In exemplo sit E F 6, & C F 8: erit C H E subtensa, 10; & H E, sinus rectus D E, peripherie dimidia per sepiam septima bujus 5: quadratum enim E F est 36, quadratum C F 64; horum aggregatum est 100, & radix ejus 10, pro subtensa C H E: Itaque H E vel H C est talium 5.

13. Sinus rectus peripherie in circuli quadrante, media proportione est ad semiradium, & sinum versum arcus dupli.

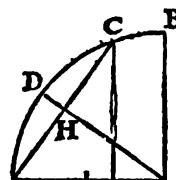
Esto in diagrammate datus arcus E D, ad quem duplus sit E C: dico A G semiradium, esse ad H E sinum rectum arcus D E; ut H E ad E F, sinum versum arcus dupli E C. Triangula enim A H E, & E F C similia sunt, ob rectos angulos ad F & H per sepiam bujus, communem ad E. Itaque latera eosdem angulos concidentia per quartam sexti elementorum sunt proportionalia. Quare ut A E latus recto appositum, ad latus C E recto oppositum; ita E H latus minus rectum ambiens, ad E F latus minus rectum ambiens. Sed ut A E ad C E; ita A G semiradius ad H E semissim subtensa, per decimam quintam quinti elementorum. Ergo ut A G ad H E; ita H E ad E F, quod erat demonstrandum.

Π Ο.

Geometriae Triangulorum Liber I.

Π Ο Ρ Ι Σ Μ Α Τ Α *duo.*

Itaque semiradio dato, & cujusvis peripheriae sinu recto, datur etiam sinus versus peripheriae duplae : Nam ut semiradius se habet ad sinum rectum peripheriae datum; ita sinus rectus peripheriae datus, ad sinum versus duplae.

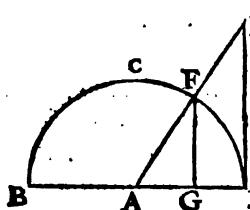


In exemplo, sit AG semiradius 5, & HE 6; erit EF 7 paulo plus. Nam ut AG 5 ad HE 6; ita HE 6 ad EF 7 paulo plus, sinum versus CE peripheriae duplae hinc verò rectos sinus AF & FC invenire non est difficile, undecima & octava bujus hoc indicantibus.

Dato vero semiradio, & sinu cuiuscunque peripheriae verso, invenitur & sinus rectus peripheriae dimidiae: factum enim à semiradio per datæ peripheriae sinum versus, & quatur sinui recto peripheriae dimidiae.

In eodem exemplo, detur AG semiradius 5; & FE sinus versus peripheriae CE 7: erit HE sinus rectus peripheriae dimidia 6 fere. Nam ut AG 5 ad HE; ita HE est ad FE 7. Factus verò ab AG 5, per FE 7, scilicet 35, est aequalis facto per se, per trigeminam septimi Euclidis. Quare bujus radix quadrata 6 fere, HE sinus recto peripheriae dimidia competit.

De quantitate rectarum extra circuli peripheriam.



E 14. Extra circuli peripheriam consideratur recta peripheriam tangens.

Talis est recta ED, tangit enim peripheriam FD.

15. Tangens peripheriae est recta linea, extremo diametri perpendicularis, in radium per arcus terminum continuatum; ipsi arcui & reliquo ad semicirculum competens.

Ita in premissa figura, tangens ED est perpendicularis extremo diametri BA D, in radium AF continuatum per arcus terminum F: competens arcui FD, & reliquo ad semicirculum BCF.

16. Tangens peripheriae se habet ad radium; ut peripheriae sinus rectus ad sinum rectum complementi.

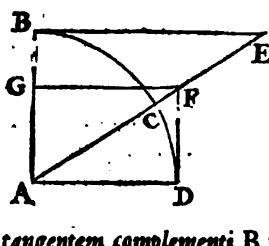
Esto in praecedenti figura recta ED, tangens peripherie FD; & ejusdem rectus sinus FG, complementi AG: radius AD. Dico rationem ED ad AD esse, ut FG ad AG. Triangula enim AFG, & AED, sunt equiangula, ob rectos angulos ad D & G, communem ad A. Itaque per quartam sexii elementorum latera habent proportionalia.

Π Ο Ρ Ι Σ Μ Α.

Quare sinu recto peripheriae cujusvis, & complementi cognito, ejusdem tangens non latebit. Nam ut rectus sinus complementi peripheriae se habet ad sinum rectum ipsius peripheriae: ita radius ad tangentem ejusdem.

In exemplo, sit AG 6, FG 8, & AD 10; erit ED 13 paulo plus. Nam ut 6 ad 8; ita 10 ad 13 paulo plus.

17. Radius media proportione est ad tangentes peripheriae, & complementi.



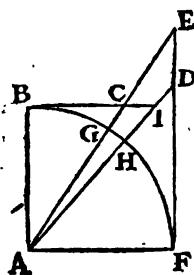
Esto recta FD, tangens peripherie DC; complementi vero BC, tangens EB: radius AB, vel AD radium, media proportione esse ad DF & BE, tangentes peripheriarum DC & BC. Triangula enim ADF (vel AGF per trigeminam quartam primi elementorum) & ABE sunt equiangula, ob rectos angulos ad B & D (vel G) communem ad A. Itaque per quartam Sexii elementorum, ut GA, tangens peripheria CD, ad GF radium: ita AB radius, ad BE tangentem complementi BC.

Π Ο Ρ Ι Σ Μ Α.

Quare tangentes arcuum complementorum suorum tangentibus reciprocè proportionales sunt.

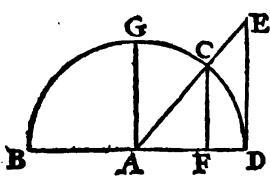
Geometriæ Triangulorum Liber I.

5



E Sint enim in adjuncto schemate arcum FH & FG, tangentes FD & FE, & complementorum BG & BH, tangentes BC & BI. Dico rationem FD ad DFE esse, ut BC ad BI. Nam per vigesimam octavi Euclidis, Similes plani sunt, inter quos unus proportionalis medius intercidit. Sed inter FD & BI, item FE & BC, unus proportionalis medius intercidit, nempe radius : semiles ergo plani sunt. Sed per penultimam definitionem septimi Euclidis similes plani latera habent proportionalia: Quare, ut FD ad FE, ita BC ad BI; quod erat demonstrandum.

De magnitudine rectarum per circuli peripheriam.



18. Per circuli peripheriam consideratur recta peripheria secans.

Talis est recta AE; secans enim peripheriam DCB in C.

19. Secans peripheria, est recta linea per peripheria terminum, in tangentem ducta; peripheria secta & reliqua ad semicirculum competens.

Ita in premissa diagrapha, secans AE ducta est per terminum peripheria DC in tangentem ED: competitque peripheria CD, & reliqua ad semicirculum BC.

20. Radius media proportione est ad peripherie sinum rectum, & secantem complementi.

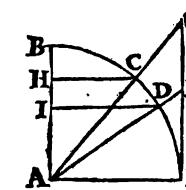
Esto in figura superiori AF, sinus rectus peripheria GC; & AE secans peripheria CD (complementis prioris ad circuli quadrantem) dico AF sinus peripheria GC esse ad AC radium, ut AD radius ad AE secantem complementi. Triangula enim AFC, & ADE, sunt aquiangula; ob rectos angulos ad F & D, communem ad A. Itaque per quartam sexti elementorum, ut AF ad AC; ita AD ad AE: quod erat demonstrandum.

ΠΟΡΙΣΜΑΤΑ δυο.

Itaque ex sinu recto cuiusvis peripheria, etiam complementi secans datur: ut enim peripheria datæ sinus rectus se habet ad radium; ita radius ad secantem complementi.

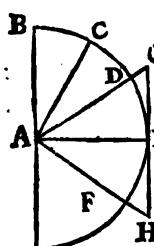
In exemplo sit AF 5, & AD 10: erit AE 20. Nam ut 5 ad 10; ita 10 ad 20, secantem AE.

Et secantes arcum complementorum rectis sinibus reciprocè proportionales sunt.



G Sint enim in adjuncta diagrapha, arcum ED & EC, secantes AF & AG; complementorum vero sinus AI & AH: Dico AF esse ad AG; ut AI ad AH. Nam per 20 octavi Euclidis: Similes plani sunt inter quos unus proportionalis medius intercidit. Sed inter secantes peripheria, & complementorum sinus, radius est medius proportionalis: quare AF, AI, item AG, AH, similes plani sunt. Sed per penultimam definitionem septimi Euclidis, Similes plani latera habent proportionalia: Ergo, ut AF ad AG; ita AI ad AH; quod erat demonstrandum.

21. Secans arcus æqualis est Tangenti dati, & semissis complementi.



Esto arcus DE, secans AG, tangens EH: Complementi vero arcus BD; & semissis BC (vel æqualis ei EF) tangens EH. Dico secantem AG, æqualem esse EG tangentis arcus dati, & EH semissis complementi. Angulus enim GAH, est æqualis angulo CAE ex thesi; angulus vero EAH est æqualis angulo BAC. Itaque angulus EHA, vel GH A, est prioris complementum, per trigesimal secundam primi elementorum: & proinde æqualis angulo CAE. Quare cum in Triangulo GAH, anguli ad A & H æquentur; manifestum est latera GA & GH per sextam primi elementorum etiam æquari: quod erat demonstrandum.

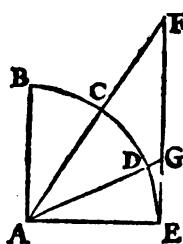
ΠΟΡΙΣΜΑ.

Quare arcus tangente, & complementi semissis simul additis, componitur dati arcus secans.

Adjiciatur enim EG, tangentis arcus ED, ad EF complementi semissis tangentem EH; componeatur HG, & æqualis ei AG secans peripheria DE.

Geometriae Triangulorum Liber II.

22. Secans arcus, circuli quadrantis semisæ minoris, cum tangentæ ejusdem, æqualis est tangenti peripheriæ datæ & semisæ complementi.



F Esto peripheria D E, secans A G; tangens E G: complementi B D, semisæ D C; tangens verò arcus C D E, recta E G F. Dico rectam A G, cum recta G E, aquari recta F E. Angulus enim A F E, æqualis est angulo B A C (nam ut B A C angulus complementum est anguli F A E: ita etiam A F E) Sed huic æqualis est F A G ex ibi: ergo anguli A F G & G A F per sextam primi elementorum aquantur. Et proinde recta A G, æqualis recta F G; & recta A G, G E simul, aquales recta F G E: quod erat demonstrandum.

N O R I S M A.

Itaque peripheriæ datæ secans, & tangens simul additæ, componunt tangentem peripheriæ, semisæ complementi autem.

Nam secans A G & tangens G E, aquantur F E tangenti peripheria E D, autæ semisæ complementi C D. Quare adiecta E G ad A G, componitur F E tangens peripheria C E.

Atque ita rectarum que in circuli peripheria, extra, & per circuli peripheriam considerantur, magnitudo demonstrata est, reliquæ est doctrina hujus usus, sequenti libro indicandus.

G E O M E T R I A T R I A N G U L O R U M L I B E R II.

De Canonis Triangulorum Syntaxi.

1. EX superioris doctrinæ fundamentis, Canonem Triangulorum componere non est difficile, certis hypothesibus ad hoc assumptis.

Geometria Triangulorum pars secunda nobis posita fuit in Syntaxi Canonis Triangulorum, ea igitur hoc libro demonstranda est.

2. Canon Triangulorum est, qui in assumpta circuli, & dimetentis mensura, omnium circuli quadrantis partium, scrupulorumque primorum, Sinus, Tangentes, & Secantes continet.

Veteres (ut supra dictum) solis subtensiæ utabantur, & proinde Triangulorum canonem appellabant eum, qui omnium semicirculis partium subtensiæ continebat. Nam verò cum præter subtensiæ & sinus, etiam tangentes, & secantes, circa circulum conferrentur, sunt & ea in Canonem Triangulorum referenda.

3. Mensura circuli assumitur partium cc lx, pars lx scrupula prima, unum scrupulum primum ix secunda potest; & ita deinceps.

Hæc circuli divisio est Ptolemei, & recentiorum Mathematicorum; valde idonea ad numerationem: inter minores enim numeros nullus ad eum multiplicipes partes habet, Vnicam, sextantem, quadrantem, tridentem, quincuncem, semissem, septuaginem, bessem, dodrantem, dextantem, decunem, & assen. Retinenda igitur est, & ad eam alia proportionaliter accommodande sunt.

4. Dimetens circuli statuitur particularum 20000000.

Ptolemaeus diametrum assumit particularum 120: Arzabel 300. Neoterici 20000000 particularum eam statuant: qua mensura retinenda est; nam cum plurimum particularum sit, plenus diameter secatur, & proinde à multis subdivisionibus logista liberatur.

5. Qualium dimetens statuitur particularum 20000000, talium latus sexanguli circulo inscripti est 10000000.

Nam per 15 quarti elementorum latus sexanguli circulo inscripti est æquale radio. Radius autem dia-

Geometriæ Triangulorum Liber II.

diametri semissis est, quare dimidiaea diametri mensura 20000000, datur radius, & aquale ei Sexanguli latus, particularum 10000000.

6. Trianguli, 17320508 ferè.

Nam per 12 decimiertii Euclidis, Latus Trianguli circulo inscripti potentia est triplum radii: Radius autem est particularum, 10000000; ergo potentia ejus triplicata est particularum 30000000000000, & latus ejus 17320508 ferè.

7. Quadranguli 14142136.

Per sextam enim quarti elementorum, Recta quadrantem circuli subtendens, est latus quadranguli circulo inscripti: potest autem ea per penultimam primi elementorum duplum radii. Itaque potentia quadranguli est 2000000000000000: & ejus latus 14142136.

8. Decanguli 6180430.

Nam per nonam decimiertii Euclidis, decanguli latus, est segmentum minus recta linea extrema & media ratione secta, latus sexanguli & decanguli simul mensurantis. Itaque per undecimam secundi elementorum ablato semiradio 5000000, ex quadrato latere radii & semiradii aggregato 11180340: relinquitur decanguli latus 6180430.

9. Quinquanguli 11755704 ferè.

Nam per decimam decimiertii Euclidis, Latus quinquanguli in circulum inscripti, potest latus sexanguli & decanguli. Sed sexanguli latus est particularum 10000000, per quintam hujus: decanguli 6180430 per premissam. Itaque per penultimam primi elementorum Quinquanguli latus est 11755704 ferè.

10. Quindecanguli 4158234 ferè.

Nam per decimam sextam quarti elementorum, recta inscripta inter basim Trianguli & Quinquanguli, ab eodem punto in circulum ducti est latus Quindecanguli. Atqui talis est D E in adjuncta figura, inscripta inter basim Trianguli D H, & Quinquanguli E G, à B eodem punto in circulum ducti: est ergo latus Quindecanguli. Hujus porro magnitudo investigatur hoc modo: datur D K H latus Trianguli per 6 hujus 17320508 ferè, & E L G Quinquanguli latus per premissum 11755704 ferè. Itaque per 7 primi Triangulorum D K est 8660254; E L 5877852, sinus recti peripheriarum FD & FE: & differentia eorum DL 2782402. Per 8 vero ejusdem A K est 5000000; A L 8090170 sinus recti complementorum CD & CE: & differentia eorum K L vel I E 3090170. Quare cum in Triangulo D I E rectangulo ad I, detur latus D I 2782402, & I E 3090170: per penultimam primi elementorum latus D E Quindecanguli est particularum 4158234 ferè; quod erat demonstrandum.

11. Si Trianguli, Quadranguli, Quinquanguli, Sexanguli, Decanguli, & tandem ipsius Quindecanguli laterum semisses assumantur, ut angulorum dimidiorum sinus: & ex his complementorum singulorum, semissimumque sinus continuè investigentur; & contra, totus sinuum Canon hac inductione componitur.

Sint inscripta laterum
suprà inventa.

Affumanturque horum semisses,
ut Angulorum dimidiorum
sinus per septimam pri-
mi Triangulorum.

Trianguli	120.	17320508 per	6	Partium	60	8660254.
Quadranguli	90.	14142196 per	7		45	7071068.
Quinquanguli	72.	11755704 per	9 hujus.		36	5877852.
Sexanguli,	60.	10000000 per	5		30	5000000.
Decanguli, &	36.	6180430 per	8		18	3090170.
Quindecanguli	24.	4158234 per	10		12	2079117.

Dico ex harum peripheriarum sinibus datis, reliquarum quadrantis peripheriarum sinus datum iri:
Si continuè harum complementorum, semissimumque sinus determinentur, & contra. Elementum veritatis

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rati sua tamen aliam non desiderat, quād inductionem ab experientiā factam, que in hoc generē sufficit, cūm numeri sensibus subjecti sint. Assumatur igitur exempli gratiā arcus partium 12, ejusque sinus 2079117; adhibeatūque presentis elementi methodus, binc sequentium peripheriarum sinus dabuntur.

Continuae semiſſes ex periph. partium 12 deducitæ.	& earum sinus per 12 vel 13 primi hujuſ.	& comple-menta,	& sinus per 8 pri-mi bujuſ.
6	1045285.	66	9135455.
3	523360.	55 30	8241262.
1 30	261769.	72 45	9550199.
0 45	130896.	50 15	7688418.
		66 45	9187912.
barumque com- plementa.	& sinus per 8 pri-mi bujuſ.	iterumque semiſſes bo- rum,	& sinus per 12 vel 13 primi bujuſ.
84	9945219.	33	5446390.
87	9986295.	16 30	2840153.
88 30	9996573.	8 15	1434926.
89 15	9999143.	27 45	4656145.
& horum se- miſſes,	& sinus per 12 vel 13 primi bujuſ.	& comple-menta,	& sinus per 8 pri-mi bujuſ.
42	6691306.	57	8386706.
21	3583679.	73 30	9588197.
10 30	1822355.	81 45	9896514.
5 15	915016.	62 15	8849876.
43 30	6883546.		
21 45	3705574.	borumque semiſſes,	& sinus per 12 vel 13 primi bujuſ.
44 15	6977905.	28 30	4771588.
barumque com- plementa,	& sinus per 8 pri-mi bujuſ.	14 15	2461533.
48	7431448.	36 45	5983246.
69	9339804.	& comple-menta,	& sinus per 8 pri-mi bujuſ.
79 30	9832549.	61 30	8788111.
84 45	9958049.	75 45	9692309.
46 30	7253744.	53 15	8012538.
68 15	9288096.		
45 45	7160319.	& semiſſes per peripe- riae 61, 30.	& sinus ejus per 12 vel 13 primi bujuſ.
rurſusque borum semiſſes,	& sinus per 12 vel 13 primi bujuſ.	30 45	5112931.
24	4067366.	bujusque comple- mentum,	& sinus per 8 pri-mi bujuſ.
34 30	5664062.		
17 15	2965416.	59 15	8594064.
39 45	6394390.		
23 15	3947439.		

His verò sinibus inventis assumendum quoque est complementum arcus partium 12, nempe 78; & inde simili inductione semiſſum peripheriarum, complementorū que sinus continuè investigandi sunt. Quae ratio si non modo in bujuſ peripheria ſinu, ſed & reliquis ſupra inventis ſervetur, tandem maxima pars Canonis absolvetur.

Ceterum cūm ad Canonem complendum etiam prioris ſcrupuli & ſequentium aliquot ſinus defiderentur, ſupereft ut quomodo ex bujuſ Theorematiſ methodo, & ii investigatingi ſint, paucis oſtendamus. Assumatur igitur ſinus partium 0, 45'. ſupra inventus 130896: adhibuitq; inductione ſuperiori bujuſ semiſſes continuè investigatingur per 12 vel 13 primi Triangul. Ita ſequentium peripheriarum ſinus inveniuntur.

$$\begin{array}{ll} 22' & 36' \\ 11 & 15 \end{array} \quad \begin{array}{l} 65449 \\ 32724 \end{array}$$

Porro cūm ex his ſinibus apparet eō uſque pervenire nos, ubi recta & curva differentia ſenſum pro- fuf evadit, tanquam una linea factarum, nullus error committetur, ſi aqua ratione reliquis peripheriis 22' 36' minoribus ſinus rectus ejus 65449 accommodetur. Ita enim ſinus ſcrupuli unius dabatur 2909 ſerè, & ſcrupulorum 15', 43632; & ita deinceps. Ex his verò ſinibus ſinuum Canon perficietur. Si diplorum arcuum & complementorum ſinus per decimam tertiam primi Triangulorum investigatingur: & ex

Geometriæ Triangulorum Liber II.

9

ex iis rursus semisum complementorumunque continuè; dum totus sinuum Canon absolutus fuerit.

Hac est sinuum Canonis condendi ratio, qua cùm ex superioris libri elementis deducta sit, ampliori demonstratione non est opus.

12. Ductis vero singulis totius quadrantis sinibus in radium, planisq; sigillatim in sinus complementorum divisis, dantur singulæ totius circuli quadrantis peripheriarum tangentes, totusque tangentium Canon hac methodo completur.

Hujus elementi ratio ex decimasexta primi bujus manifesta est. Nam per eandem Tangens peripheria se habet ad radium; ut peripheria sinus rectus ad sinum complementi. Itaque cùm sinuum Canon ex superiori doctrina compositus sit, componetur etiam tangentium Canon: multiplicatis singulis totius quadrantis sinibus in radium, planisque horum sigillatim in complementorum suorum sinus divisis. Exempli gratia, datur sinus partium 30, 5000000, & complementi sui 8660254: ergo tangentis partium 30 erit 5773502. Nam ut 8660254 ad 5000000; ita 10000000 se habet ad 5773502.

13. Secantiuni Canon componitur, radii quadrato in singulos totius circuli quadrantis sinus diviso, initio à sinuum Canonis fine facto.

Nam per vigesimam primi bujus, Radius media proportione est ad peripheria sinum rectum, & secantem complementi. Itaque assumptis singulis totius quadrantis sinibus à fine Canonis, divisisque iis in radii quadratum; dantur totius quadrantis secantes: & proinde earum Canon hac viâ completur. In exemplo superiori, datur partium 30 sinus rectus 5000000, & quadratum radii 1000000000000000: ergo secans partium 60 assumpta scilicet peripheria complementi est 20000000. Nam ut 5000000 ad 10000000; ita 10000000 ad 20000000.

Atque hac quidem methodus est Constructionis Canonis Sinuum, Tangentium & Secantium, in quā tamen spōmē à nobis omissa sunt compendia superioris libri Theorematibus 9, 21 & 22 demonstrata. Nam cùm integer Triangulorum Canon ad manum esset, Sinuum quidem à præstantissimo Mathematico Joanne Regiomontano, Tangentium ab Erasmo Reinholdo, Secantium verò ab Ioachimo Rhetico compositus, latius ista persequi supervacuum duximus. Sufficit enim demonstrasse ex quibus fundamentis Canonis Triangul. constructione deducta, & qua methodo à præstantissimis artificibus completus sit. Reliquis est Canonis usus quem sequenti theoremate preponimus.

14. Canon Triangulorum in fronte partes circuli quadrantis, in sinistro margine, partium scrupula prima, in communi intersectione, partis scrupulique sinus, tangentes vel secantes, cum differentia 60 scrupulis secundis competente complectitur.

Canonis frons, vulgo tabula caput, suprema pars, aut transversalis margo appellatur: continetque totius circuli quadrantis partes. Sinister margo est in quo partium scrupula prima descripta sunt. Communi intersectio, vel angulus est, in quo descendens & transversalis ordo se mutud intersecant. Differentia verò 60 secundis scrup. competens, est excessus minoris sinus, tangentis vel secantis, super proximè majorem.

P O R I S M A T A duo.

Itaque assumptæ partis, & primi scrupuli sinus, tangens, vel secans in Canone est, quæ in angulo communi partis assumptæ, & scrupuli primi continetur: & contra.

In exemplo, sinus partium 23 & scrupulorum primorum 28 est 3982155: Talis enim in angulo communi sinuum canonis exhibetur. Viceversa 3982155 sinus est partium 2328: Inventus enim sinus in Canone, partes 23 in fronte, scrupula verò prima 28 ostendit.

Parti vero & scrupulis primis, etiam secundis adhærentibus pars proportionalis differentiæ (quæ l. x scrupulis secundis competit) sinui, tangenti, vel secanti proximè minori addita, assumptæ peripheria sinum, tangentem, vel secantem componit; & contra.

Exempli gratiâ, sinus partium 23, scrup. pr. 28, scrup. sec. 30, est 3983489. Nam proximè minor sinus in Canone invenitur, 3982155; & differentia scrupulis 60 secundis competens est 2668: ergo proportionalis pars 30 secundis tribuenda est 1334 (Nam per autem regulam, ut 60 ad 2668; ita 30 ad 1334) hac autem sinui minori 3982155 adjecta componit 3983489, sinum peripheria 23-28-30 quartum. Viceversa peripheria sinus 3983489, ex sinuum Canone invenitur partium 23-28-30. Nam sinus proximè minor 3982155, competit arcui partium 23-28. Differentia vero bujus sinus & precedentis dati est 1334: cui congruum 30 scrupula secunda, (Nam ut 2668 differentia sexaginta scrupulis secundis competens, ad scrupula 60 secunda: ita 1334 ad 30 scrupula secunda) Itaque his ad arcum 23-28 proximè minorem adjectis, componitur peripheria partium 23-28-30, sinui proposito 3983489, correspondens. Et hic quidem Canonis usus. Nam ipsum Canonem subiiciamus.

Canon Sinuum Tangentium & Secantium.

O	Sinus	Tangens	Secans	
0	100000.00	Infinitum.	100000.00	Infinitum.
1	29.09	99999.99	29.09	343774667.38
2	58.18	99999.98	58.18	171887319.15
3	87.27	99999.96	87.27	114591529.94
4	116.36	99999.93	116.36	85943680.48
5	145.44	99999.89	145.44	68754959.63
6	174.53	99999.84	174.53	57295721.34
7	203.61	99999.79	203.61	49110600.28
8	232.71	99999.73	232.71	42971757.06
9	261.80	99999.66	261.80	38197099.08
10	290.89	99999.58	290.89	34377370.74
11	319.98	99999.49	319.98	31152136.71
12	349.06	99999.39	349.07	28647773.40
13	378.15	99999.28	378.16	26444079.88
14	407.24	99999.17	407.25	24555198.33
15	436.33	99999.05	436.33	21918166.36
16	465.42	99998.93	465.42	21485762.18
17	494.51	99998.78	494.51	20221874.99
18	523.60	99998.63	523.60	19098418.64
19	552.68	99998.47	552.69	18093219.83
20	581.77	99998.30	581.78	17188539.93
21	610.86	99998.13	610.87	16370019.10
22	639.95	99997.95	639.96	15625908.37
23	669.04	99997.76	669.05	14946501.08
24	698.13	99997.56	698.14	14332712.17
25	727.21	99997.35	727.23	13750744.68
26	756.30	99997.13	756.32	13221850.86
27	785.39	99996.91	785.41	12732133.65
28	814.48	99996.68	814.50	12277395.54
29	843.57	99996.44	843.60	11854018.02
30	872.65	99996.19	872.69	11458865.01
31	901.74	99995.93	901.71	11089201.13
32	930.83	99995.66	930.87	107421648.38
33	959.91	99995.39	959.96	10417094.48
34	989.00	99995.11	989.05	10110690.24
35	1018.09	99994.82	1018.14	9821794.26
36	1047.18	99994.52	1047.24	9548947.52
37	1076.27	99994.21	1076.33	929048.73
38	1105.35	99993.89	1105.41	9046333.57
39	1134.44	99993.56	1134.51	8814924.39
40	1163.53	99993.23	1163.61	8594160.86
41	1192.61	99992.89	1191.70	8384330.67
42	1221.70	99991.54	1221.79	8184704.11
43	1250.79	99991.18	1250.88	7994343.99
44	1279.87	99991.81	1279.98	7812634.20
45	1308.96	99991.43	1309.07	7639000.93
46	1338.05	99991.04	1338.17	7472916.51
47	1367.13	99990.65	1367.26	7313899.10
48	1396.22	99990.25	1396.35	7161507.01
49	1425.30	99989.84	1425.45	7015334.61
50	1454.39	99989.42	1454.54	6875008.72
51	1483.48	99988.99	1483.64	6740185.43
52	1512.56	99988.55	1512.73	6610547.27
53	1541.65	99988.11	1541.83	6485800.75
54	1570.73	99987.66	1570.93	6365674.12
55	1599.81	99987.20	1600.02	6240915.37
56	1618.90	99986.73	1629.12	6138390.52
57	1647.99	99986.25	1658.11	6030581.99
58	1687.07	99985.76	1687.21	5926587.21
59	1716.16	99985.27	1716.31	5826117.35
60	1745.24	99984.77	1745.51	5728906.16

Canon Sinuum, Tangentium & Secantium.

	Sinus	Tangens	Secans	
0	1745.24	99984.77	1745.51	5718996.16
1	1774.32	99984.86	1774.60	5635018.96
2	1803.41	99984.74	1803.70	5544151.67
3	1832.49	99983.81	1832.80	5456130.03
4	1861.58	99984.67	1861.90	5370858.75
5	1890.66	99983.18	1891.00	528810.91
6	1919.74	99981.57	1920.10	5208067.26
7	1948.83	99981.01	1949.20	5130819.66
8	1977.91	99980.44	1978.30	5054850.59
9	2006.99	99979.86	2007.40	4981572.64
10	2036.08	99979.57	2036.50	4910388.06
11	2065.16	99978.67	2065.60	4841208.41
12	2094.24	99978.06	2094.70	4773950.14
13	2123.32	99977.45	2123.80	4708334.80
14	2152.41	99976.83	2153.81	4644886.80
15	2181.49	99976.20	2182.01	4581935.13
16	2210.57	99975.56	2211.11	452614.07
17	2239.65	99974.91	2240.21	4463859.56
18	2268.73	99974.35	2269.32	4406611.33
19	2297.81	99973.59	2298.42	4350812.16
20	2326.90	99973.92	2327.53	4296407.73
21	2355.98	99974.24	2356.63	4248346.39
22	2385.06	99971.55	2385.74	4191576.99
23	2414.14	99970.85	2414.84	4141058.76
24	2443.21	99970.14	2443.95	4091761.16
25	2472.30	99969.43	2473.05	4043533.75
26	2501.38	99968.71	2502.16	3996546.05
27	2530.46	99967.98	2531.27	3950589.46
28	2559.54	99967.24	2560.38	3905677.11
29	2588.62	99966.49	2589.48	3861773.81
30	2617.69	99965.73	2618.39	3818845.93
31	2646.77	99964.96	2647.70	3776861.30
32	2675.85	99964.19	2676.81	3737789.17
33	2704.93	99963.41	2705.92	3695600.11
34	2734.01	99962.62	2735.03	3656265.91
35	2763.09	99961.81	2764.14	3617759.61
36	2792.16	99961.01	2793.25	3580055.33
37	2821.24	99960.19	2822.36	3543128.25
38	2850.32	99959.36	2851.43	3506954.58
39	2879.40	99958.53	2880.59	3471911.50
40	2908.47	99957.69	2909.70	3436777.09
41	2937.55	99956.84	2938.82	3402730.59
42	2966.62	99955.98	2967.93	3369350.89
43	2995.70	99955.11	2997.05	3336619.45
44	3024.78	99954.24	3026.16	3304517.57
45	3053.85	99953.36	3055.28	3273026.57
46	3082.93	99952.47	3084.39	3242159.46
47	3112.00	99951.57	3113.51	3211809.88
48	3141.08	99950.66	3142.63	3182051.60
49	3170.15	99949.74	3171.74	315839.16
50	3199.22	99948.81	3200.86	3124158.08
51	3228.30	99947.88	3229.98	3095992.80
52	3257.37	99946.94	3259.10	3068330.70
53	3286.44	99945.99	3288.22	3041158.08
54	3315.51	99945.03	3317.34	3014461.89
55	3344.59	99944.06	3346.46	2988219.86
56	3373.66	99943.08	3375.38	2968449.95
57	3402.73	99942.09	3404.71	2937110.55
58	3431.81	99941.09	3433.83	2912300.47
59	3460.88	99940.09	3462.97	2887708.88
60	3489.95	99939.08	3492.08	2863625.33

Canon Sinuum, Tangentium & Secantium.

2	Sinus	Tangens	Secans	
0	3499.95	99939.08	3492.08	2863625.33
1	3519.02	99938.06	3521.20	2839939.09
2	3548.09	99937.03	3550.33	2816642.18
3	3577.16	99935.99	3579.45	2793723.33
4	3606.23	99934.91	3608.18	2771173.99
5	3635.30	99933.90	3637.71	2748985.38
6	3664.37	99932.84	3666.83	2727148.61
7	3693.44	99931.77	3695.96	2705655.68
8	3722.51	99930.69	3725.09	2684498.43
9	3751.58	99929.60	3754.22	2663669.04
10	3780.65	99928.51	3783.35	2643159.96
11	3809.71	99927.40	3812.48	2622963.84
12	3838.78	99926.29	3841.61	2603073.78
13	3867.85	99925.17	3870.74	2583482.27
14	3896.91	99924.04	3899.88	2564183.23
15	3925.98	99923.90	3929.01	2545169.95
16	3955.05	99921.75	3958.14	2526436.15
17	3984.11	99920.60	3987.88	2507975.68
18	4013.18	99919.44	4016.41	2489782.62
19	4042.24	99918.27	4045.55	2471851.19
20	4071.31	99917.09	4074.69	2454175.78
21	4100.37	99915.90	4103.83	2436750.95
22	4129.44	99914.70	4132.96	2419571.40
23	4158.50	99913.49	4162.10	2402631.99
24	4187.57	99912.28	4191.84	2385927.72
25	4216.63	99911.06	4220.38	2369453.72
26	4245.69	99909.83	4249.53	2331305.25
27	4274.75	99908.59	4278.66	2317177.72
28	4303.82	99907.34	4307.81	2311366.65
29	4332.88	99906.08	4336.95	2305767.67
30	4361.94	99904.82	4366.09	2290376.55
31	4391.00	99903.55	4395.24	2275189.16
32	4420.06	99902.27	4424.38	2260201.48
33	4449.12	99900.98	4453.53	2245409.59
34	4478.18	99899.68	4482.68	2230809.67
35	4507.34	99898.37	4511.82	2216398.02
36	4536.30	99897.05	4540.97	2202171.00
37	4565.36	99895.73	4570.12	2188125.10
38	4594.42	99894.40	4599.27	2174156.87
39	4623.47	99893.06	4623.42	2163562.96
40	4651.53	99891.71	4657.57	2147040.10
41	4681.59	99890.35	4686.73	2133685.11
42	4710.64	99888.98	4715.88	2120494.88
43	4739.70	99887.61	4745.03	2107466.37
44	4768.76	99886.23	4774.19	2094196.63
45	4797.81	99884.84	4803.34	2081882.76
46	4826.87	99883.44	4832.50	2069321.96
47	4855.92	99882.03	4861.66	2056911.47
48	4884.98	99880.61	4890.82	2044648.61
49	4914.03	99879.18	4919.97	2032130.75
50	4943.08	99877.75	4949.13	2020555.35
51	4972.14	99876.31	4978.29	2008719.89
52	5001.19	99874.86	5007.46	1997031.95
53	5030.24	99873.40	5036.62	1985459.12
54	5059.29	99871.93	5065.78	1974039.10
55	5088.35	99870.41	5094.95	1963739.59
56	5117.40	99868.97	5124.11	1951558.37
57	5146.45	99867.48	5153.28	1940513.27
58	5175.50	99865.98	5182.44	1929592.17
59	5204.55	99864.47	5211.61	1918792.98
60	5233.60	99863.95	5240.78	1908113.67

Canon Sinuum, Tangentium & Secantium.

3	Sinus	Tangens	Secans	
0	5233.60	99862.95	5240.78	1908113.67
1	5262.64	99861.42	5269.95	1807552.26
2	5291.69	99859.89	5299.12	1887106.80
3	5320.74	99858.35	5328.29	1876775.39
4	5349.79	99856.80	5337.46	1866556.18
5	5378.83	99855.24	5386.03	185647.34
6	5407.88	99853.67	5415.81	1846447.09
7	5436.93	99851.09	5444.98	1836533.70
8	5465.97	99850.50	5474.16	1826765.44
9	5495.02	99848.91	5503.33	1817080.67
10	5524.06	99847.31	5532.51	1807497.74
11	5553.11	99845.70	5560.69	1798015.05
12	5582.15	99844.08	5590.87	1788631.04
13	5611.19	99842.45	5620.05	1779344.17
14	5640.24	99840.81	5649.23	1770152.94
15	5669.28	99839.16	5678.41	1761055.88
16	5698.32	99837.51	5707.59	1752051.55
17	5727.36	99835.85	5736.78	1742138.54
18	5756.40	99834.18	5765.96	1734315.46
19	5785.44	99832.50	5795.15	1725580.95
20	5814.48	99830.81	5824.34	1716933.69
21	5843.52	99829.11	5853.51	1708372.38
22	5872.56	99827.41	5882.71	1699895.74
23	5901.60	99825.70	5911.90	1691502.51
24	5930.64	99823.98	5941.09	1683191.48
25	5959.67	99822.25	5970.29	1674961.44
26	5988.71	99820.51	5999.48	1666813.30
27	6017.75	99818.76	6028.67	1658739.62
28	6046.78	99817.01	6057.87	1650745.55
29	6075.82	99815.25	6087.66	1642827.89
30	6104.85	99813.48	6116.86	1634985.55
31	6133.89	99811.70	6145.46	1627217.44
32	6162.92	99809.91	6174.05	1619523.53
33	6191.96	99808.11	6203.86	1611899.79
34	6220.99	99806.30	6233.06	1604348.19
35	6250.02	99804.49	6261.26	1596866.74
36	6279.05	99802.67	6291.47	1589454.48
37	6308.08	99800.84	6320.67	1582110.45
38	6337.11	99799.00	6349.88	1578833.71
39	6366.14	99797.15	6379.08	1567623.33
40	6395.17	99795.39	6408.29	1560478.41
41	6424.20	99793.43	6437.50	1553198.06
42	6453.23	99791.56	6466.71	1546381.41
43	6482.26	99789.68	6495.92	1539427.60
44	6511.29	99787.79	6525.13	1531535.80
45	6540.31	99785.89	6554.35	1525705.17
46	6569.34	99783.98	6583.56	1518934.90
47	6598.36	99782.06	6613.78	1513322.40
48	6627.39	99780.14	6641.99	1505572.27
49	6656.41	99778.21	6671.21	1498928.36
50	6685.44	99776.27	6700.43	1492441.70
51	6714.46	99774.32	6729.65	1481961.55
52	6743.48	99772.36	6758.87	1479537.18
53	6772.51	99770.39	6788.09	1473167.87
54	6801.53	99768.42	6817.32	1466852.93
55	6830.55	99766.44	6846.54	1460591.63
56	6859.57	99764.45	6875.77	1454383.33
57	6888.59	99762.45	6904.99	1448227.31
58	6917.61	99760.44	6934.22	1442122.97
59	6946.63	99758.41	6963.45	1436069.61
60	6975.65	99756.40	6992.68	1430066.63

Canon Sinuum, Tangentium & Secantium.

4	Sinus	Tangens	Secans	
0	6975.65	99756.40	6992.68	1430066.63
1	7004.66	99754.37	7021.91	1424113.37
2	7033.68	99752.33	7051.13	1418209.24
3	7062.70	99750.28	7080.38	1412353.63
4	7091.71	99748.22	7100.61	1406545.93
5	7120.73	99746.15	7138.85	1400985.56
6	7149.74	99744.07	7168.09	1395071.94
7	7178.76	99741.99	7197.33	1389404.51
8	7207.77	99739.90	7226.57	1383782.70
9	7236.78	99737.80	7255.81	1378205.98
10	7265.80	99735.69	7285.05	1372673.79
11	7294.81	99733.57	7314.30	1367185.60
12	7323.82	99731.44	7343.54	1361740.89
13	7352.83	99729.31	7372.79	1356339.15
14	7381.84	99727.17	7402.03	1350979.86
15	7410.85	99725.02	7431.28	1345662.53
16	7439.86	99723.86	7460.53	1340386.67
17	7468.87	99720.69	7489.79	1335151.79
18	7497.87	99718.51	7519.04	1329057.41
19	7526.88	99716.32	7548.29	1324803.07
20	7555.89	99714.13	7577.55	1319088.30
21	7584.89	99711.93	7606.80	1314612.66
22	7613.90	99709.72	7636.06	1309575.68
23	7642.90	99707.50	7665.32	1304576.93
24	7671.90	99705.27	7694.58	1299615.98
25	7700.91	99703.03	7723.84	1294952.40
26	7729.91	99700.79	7753.11	1289805.77
27	7758.91	99698.54	7782.37	1284955.66
28	7787.91	99696.38	7811.64	1281014.16
29	7816.91	99694.01	7840.90	1275363.41
30	7845.91	99691.73	7870.17	1270620.47
31	7874.91	99689.44	7899.44	1261912.46
32	7903.91	99687.15	7928.71	1261239.00
33	7932.90	99684.85	7957.98	1256599.71
34	7961.90	99682.54	7987.26	1251994.20
35	7990.90	99680.22	8016.53	1247422.12
36	8019.89	99677.89	8045.81	1242883.10
37	8048.89	99675.55	8075.09	1238376.79
38	8077.88	99673.20	8104.37	1233902.82
39	8106.87	99670.85	8133.65	1229460.85
40	8135.87	99668.49	8162.93	1215050.55
41	8164.86	99666.12	8192.21	1220671.56
42	8193.85	99663.74	8221.50	1216323.56
43	8222.84	99661.35	8250.78	1212006.22
44	8251.83	99658.95	8280.07	1207719.22
45	8280.82	99656.55	8309.36	1203462.23
46	8309.81	99654.14	8338.65	1199234.95
47	8338.80	99651.72	8367.94	1195037.05
48	8367.78	99649.29	8397.23	1190868.24
49	8396.77	99646.85	8426.53	1186728.21
50	8425.76	99644.40	8455.83	1181616.67
51	8454.74	99641.94	8485.12	1178533.31
52	8483.73	99639.48	8514.42	1174477.86
53	8512.71	99637.01	8543.72	1170450.03
54	8541.69	99634.53	8573.02	1166449.53
55	8570.67	99632.04	8602.33	1162476.08
56	8599.66	99629.54	8631.03	1158529.42
57	8628.64	99627.03	8660.94	1154609.27
58	8657.62	99624.52	8689.25	1150711.36
59	8686.60	99622.00	8719.56	1146847.43
60	8715.57	99619.47	8749.87	1143005.23

Canon Sinuum, Tangentium & Secantium.

ξ	Sinus	Tangens	Secans				
0	8715.57	99619.47	8748.87	1143005.23	100381.98	1147371.32	60
1	8744.55	99616.93	8778.18	1139188.49	100384.54	1143569.16	59
2	8773.53	99614.38	8807.49	1135396.96	100387.11	1139793.20	58
3	8802.51	99611.82	8836.81	1131630.40	100389.69	1136040.21	57
4	8831.48	99609.26	8866.12	1127888.55	100392.28	1132313.93	56
5	8860.46	99606.69	8895.44	1124171.17	100394.87	1128610.13	55
6	8889.43	99604.11	8924.76	1120478.03	100397.47	1124931.56	54
7	8918.40	99601.51	8954.08	1116808.88	100400.08	1121276.99	53
8	8947.38	99598.92	8983.41	1113163.50	100402.70	1128645.17	52
9	8976.35	99596.31	9012.73	1109541.64	100405.33	1114038.90	51
10	9005.32	99593.69	9042.06	1105943.10	100407.97	1110454.92	50
11	9034.29	99591.07	9071.38	1102307.63	100410.61	1106894.03	49
12	9063.26	99588.44	9100.71	1098815.01	100413.26	1103355.99	48
13	9092.23	99585.80	9130.04	1095285.04	100415.92	1099840.59	47
14	9121.19	99583.15	9159.38	1091777.49	100418.59	1096347.61	46
15	9150.16	99580.49	9188.71	1088292.14	100421.27	1091876.84	45
16	9179.13	99577.82	9218.04	1084828.80	100423.96	1089480.07	44
17	9208.09	99575.15	9247.38	1081387.24	100426.66	1086001.09	43
18	9237.06	99573.47	9276.72	1077967.27	100429.37	1082595.69	42
19	9266.02	99569.78	9306.05	1074568.68	100432.08	1079811.68	41
20	9294.99	99567.08	9335.40	1071191.26	100434.80	1075848.84	40
21	9323.95	99564.37	9364.74	1067834.84	100437.53	1072506.99	39
22	9352.91	99561.65	9394.09	1064499.19	100440.27	1069285.92	38
23	9381.87	99558.92	9423.44	1061184.14	100443.02	1065885.45	37
24	9410.83	99556.19	9452.13	1057889.50	100445.78	1062605.38	36
25	9439.79	99553.45	9482.13	1054615.07	100448.55	1059345.53	35
26	9468.75	99550.70	9511.48	1051360.67	100451.33	1056105.70	34
27	9497.71	99547.94	9540.84	1048126.11	100454.11	1052885.72	33
28	9526.66	99545.17	9570.19	1044911.22	100456.90	1049685.41	32
29	9555.62	99542.40	9599.55	1041715.81	100459.70	1046504.68	31
30	9584.58	99539.62	9628.90	1038539.71	100462.51	1043343.05	30
31	9613.53	99536.83	9658.26	1035381.74	100465.33	1040100.66	29
32	9642.48	99534.03	9687.63	1032244.73	100468.16	1037077.23	28
33	9671.44	99531.22	9716.99	1029125.51	100470.99	1033972.59	27
34	9700.39	99528.40	9746.35	1026034.90	100473.83	1030886.56	26
35	9729.34	99525.57	9775.72	1022942.76	100476.68	1027818.99	25
36	9758.29	99522.74	9805.09	1019878.90	100479.54	1024769.71	24
37	9787.24	99519.90	9834.46	1016833.16	100482.41	1021738.55	23
38	9816.19	99517.05	9863.83	1013805.39	100485.29	1018725.36	22
39	9845.14	99514.19	9893.20	1010795.42	100488.18	1015739.98	21
40	9874.08	99511.32	9922.57	1007803.11	100491.08	1012752.24	20
41	9903.03	99508.44	9951.95	1004828.28	100493.99	1009792.00	19
42	9931.97	99505.55	9981.33	1001870.80	100496.90	1006849.09	18
43	9960.92	99502.66	10010.71	998930.50	100499.81	1003923.38	17
44	9989.86	99499.76	10040.09	996007.24	100502.75	1001014.70	16
45	10018.81	99496.85	10069.47	993100.88	100505.69	998122.91	15
46	10047.75	99493.93	10098.85	990211.35	100508.64	995247.87	14
47	10076.69	99491.00	10128.24	987338.23	100511.60	992389.43	13
48	10105.63	99488.06	10157.63	98481.66	100514.57	989547.44	12
49	10134.57	99485.11	10187.02	981641.40	100517.54	986721.76	11
50	10163.51	99482.17	10216.41	978817.32	100520.52	983912.27	10
51	10192.45	99479.31	10245.80	976009.17	100523.51	981118.80	9
52	10221.38	99476.34	10275.20	97317.13	100526.51	978348.24	8
53	10250.32	99473.26	10304.60	970440.75	100529.52	975179.44	7
54	10279.25	99470.27	10334.00	967680.00	100532.54	974833.27	6
55	10308.19	99467.28	10363.40	964934.75	100535.57	970101.60	5
56	10337.12	99464.38	10392.80	961204.86	100538.60	967387.30	4
57	10366.05	99461.37	10421.20	959490.22	100541.64	964687.24	3
58	10394.99	99458.25	10451.60	956790.68	100544.69	961003.29	2
59	10423.92	99455.21	10481.01	954106.13	100547.75	959334.33	1
60	10452.85	99452.18	10510.42	951436.45	100550.82	956677.22	0

Canon Sinuum, Tangentium & Secantium.

6	Sinus	Tangens	Secans	
0	10453.85	99452.18	10510.42	951436.45
1	10481.78	99449.14	10539.83	948781.49
2	10510.70	99446.09	10569.24	936141.16
3	10539.63	99443.03	10598.66	943515.31
4	10568.56	99439.96	10618.08	940903.84
5	10597.48	99436.88	10657.50	938306.63
6	10626.41	99433.79	10686.93	935733.55
7	10655.33	99430.69	10716.34	933154.80
8	10684.25	99427.59	10745.70	930599.36
9	10713.18	99424.48	10775.19	928058.02
10	10742.10	99421.36	10804.62	925530.35
11	10771.02	99418.23	10834.05	923016.27
12	10799.94	99415.09	10863.48	920515.64
13	10828.85	99411.94	10892.91	918028.38
14	10857.77	99408.79	10921.34	915554.36
15	10886.69	99405.63	10951.78	913093.48
16	10915.60	99402.46	10981.23	910645.64
17	10944.52	99399.18	11010.66	908210.74
18	10973.43	99396.09	11040.10	905788.67
19	11002.34	99392.89	11069.54	903379.33
20	11031.26	99389.69	11098.99	900982.61
21	11060.17	99386.48	11118.44	898598.43
22	11089.08	99383.26	11137.89	896226.68
23	11117.99	99380.03	11157.34	893867.26
24	11146.89	99376.79	11186.79	891520.08
25	11175.80	99373.54	11246.25	889185.05
26	11204.71	99370.28	11275.71	886862.26
27	11233.61	99367.02	11305.17	884531.03
28	11262.52	99363.75	11334.63	882251.86
29	11291.42	99360.47	11364.09	879964.46
30	11320.32	99357.18	11393.56	877688.74
31	11349.23	99353.88	11423.03	875424.61
32	11378.12	99350.58	11452.50	873171.98
33	11407.02	99347.27	11481.97	870910.77
34	11435.92	99343.95	11511.44	868700.88
35	11464.82	99340.62	11540.91	866482.23
36	11493.71	99337.28	11570.39	864274.75
37	11522.61	99333.93	11599.87	862078.33
38	11551.51	99330.57	11619.35	859892.90
39	11580.40	99327.20	11658.83	857718.38
40	11609.29	99323.83	11688.31	855554.68
41	11638.18	99320.45	11717.80	853401.71
42	11667.07	99317.06	11747.29	851291.41
43	11695.96	99313.66	11776.78	849127.71
44	11724.85	99310.25	11806.28	847006.51
45	11753.74	99306.84	11835.78	844895.73
46	11782.63	99303.42	11865.28	842795.31
47	11811.51	99299.99	11894.78	840705.15
48	11840.40	99296.55	11924.28	838615.19
49	11869.28	99293.10	11953.78	836555.36
50	11898.16	99289.64	11983.28	834495.57
51	11927.04	99286.17	12012.79	832445.77
52	11955.93	99282.70	12042.30	830405.86
53	11984.81	99279.22	12071.81	828375.79
54	12013.68	99275.73	12101.32	826355.47
55	12042.56	99272.23	12130.84	824344.85
56	12071.44	99269.72	12160.36	822343.84
57	12100.31	99265.21	12189.88	820352.39
58	12129.19	99261.69	12219.40	818700.41
59	12158.06	99258.16	12248.93	816397.86
60	12186.93	99254.62	12278.46	814434.64

Canon Sinuum, Tangentium & Secantium.

7	Sinus	Tangens	Secans	
0	12186.93	99254.62	12278.46	814434.64
1	12215.81	99251.07	12307.99	812480.71
2	12244.68	99247.51	12337.52	810535.99
3	12273.55	99243.94	12367.05	808600.42
4	12301.41	99240.36	12396.58	806673.94
5	12321.88	99236.78	12426.12	804716.47
6	12350.15	99233.19	12455.66	802847.96
7	12389.01	99229.59	12485.20	800948.35
8	12417.88	99225.98	12514.74	799057.56
9	12446.74	99222.36	12544.29	797175.55
10	12475.60	99218.74	12573.84	795302.24
11	12504.46	99215.11	12603.39	793437.58
12	12533.32	99211.47	12632.94	791581.51
13	12562.18	99207.82	12662.49	789733.96
14	12591.04	99204.16	12692.05	787894.89
15	12619.90	99200.49	12721.61	786064.23
16	12648.75	99196.81	12751.17	784241.91
17	12677.61	99193.13	12780.73	782427.90
18	12706.46	99189.44	12810.29	780622.12
19	12735.31	99185.74	12839.86	778824.53
20	12764.16	99182.03	12869.43	777035.66
21	12793.01	99178.31	12899.00	775253.66
22	12821.86	99174.59	12918.57	773480.28
23	12850.71	99170.86	12958.15	771714.86
24	12879.56	99167.12	12987.73	769957.35
25	12908.41	99163.37	13017.31	768207.69
26	12937.25	99159.61	13046.89	766465.84
27	12966.09	99155.84	13076.48	764731.74
28	12994.94	99151.66	13106.07	763055.33
29	13023.78	99148.28	13135.66	761286.57
30	13052.62	99144.49	13165.25	759575.41
31	13081.46	99140.69	13194.84	757871.79
32	13110.30	99136.88	13224.44	756175.67
33	13139.13	99133.06	13254.04	754486.99
34	13167.97	99129.23	13283.64	752805.71
35	13196.81	99125.39	13313.24	751131.78
36	13225.64	99121.55	13341.85	749465.14
37	13254.47	99117.70	13372.46	747805.76
38	13283.30	99113.84	13402.07	746153.57
39	13312.13	99109.97	13431.68	744508.55
40	13340.96	99106.09	13461.29	742870.64
41	13369.79	99102.21	13490.91	741239.78
42	13398.62	99098.32	13520.53	739015.95
43	13427.44	99094.43	13550.15	737999.09
44	13456.27	99090.51	13579.77	736389.16
45	13485.09	99086.59	13609.40	734786.10
46	13513.93	99082.66	13639.03	733189.89
47	13541.74	99078.73	13668.66	731600.47
48	13571.56	99074.78	13698.29	730017.80
49	13600.38	99070.83	13727.93	728441.84
50	13629.19	99066.87	13757.57	726872.55
51	13658.01	99062.90	13787.21	725309.87
52	13686.83	99058.92	13816.85	723753.78
53	13715.64	99054.93	13846.50	722320.21
54	13744.45	99050.94	13876.15	720661.16
55	13773.27	99046.94	13905.80	719124.56
56	13802.08	99041.93	13935.45	717594.37
57	13830.89	99036.91	13965.10	716070.56
58	13859.70	99034.88	13994.76	714553.08
59	13888.50	99030.84	14024.42	713041.90
60	13917.31	99026.80	14054.08	711536.97

Canon Sinuum, Tangentium & Secantium.

8	Sinus	Tangens	Secans				
0	13917.31	99016.80	14094.08	711536.97	100982.76	718529.65	60
1	13946.12	99021.75	14033.74	710038.26	100980.89	717045.56	59
2	13974.92	99018.69	14113.41	708545.73	100991.03	715567.64	58
3	14003.72	99014.63	14143.08	707059.34	100991.18	714095.87	57
4	14032.52	99010.54	14179.75	705579.09	100999.34	713630.19	56
5	14061.32	99006.45	14205.43	704104.82	101003.51	711170.58	55
6	14090.12	99002.36	14232.11	703636.63	101007.69	709717.00	54
7	14118.92	98998.26	14261.79	701174.41	101011.88	708169.41	53
8	14147.72	98994.15	14291.47	699718.06	101016.07	706827.77	52
9	14176.51	98990.03	14321.15	698267.81	101020.17	705398.05	51
10	14205.31	98985.90	14350.84	696823.35	101024.48	703962.20	50
11	14234.10	98981.76	14380.53	695384.73	101028.70	702538.20	49
12	14262.89	98977.62	14410.22	693951.93	101032.93	701120.01	48
13	14291.68	98973.47	14439.91	692524.89	101037.17	699707.60	47
14	14320.47	98969.31	14469.61	691103.59	101041.42	698300.92	46
15	14349.26	98965.14	14499.31	689687.99	101045.68	696899.94	45
16	14378.05	98960.96	14529.01	688278.07	101049.95	695504.64	44
17	14406.84	98956.77	14558.71	686873.78	101054.23	694114.96	43
18	14435.63	98952.57	14588.42	685475.08	101058.51	692720.89	42
19	14464.40	98948.37	14618.13	684081.96	101062.80	691352.39	41
20	14493.19	98944.16	14647.84	682694.37	101067.10	689979.42	40
21	14521.97	98939.94	14677.55	681312.27	101071.41	688611.95	39
22	14550.75	98935.71	14707.27	679935.65	101075.73	687249.95	38
23	14579.53	98931.47	14736.99	678504.46	101080.06	685893.38	37
24	14608.30	98927.28	14766.71	677198.67	101084.40	684543.22	36
25	14637.08	98922.98	14796.44	675838.26	101088.75	683196.42	35
26	14665.85	98918.72	14826.17	67483.18	101093.11	681855.97	34
27	14694.63	98914.47	14855.90	673133.41	101097.47	680520.82	33
28	14723.40	98910.17	14885.63	671788.91	101101.84	679190.95	32
29	14752.17	98905.88	14915.36	670449.66	101106.22	677866.32	31
30	14780.94	98901.58	14945.10	669115.62	101110.61	676546.91	30
31	14809.71	98897.28	14974.84	667786.77	101115.01	675231.68	29
32	14838.48	98893.97	15004.58	666463.07	101119.42	673923.60	28
33	14867.24	98888.65	15034.33	665144.49	101123.84	672619.65	27
34	14896.01	98884.43	15064.08	663831.00	101128.27	671320.79	26
35	14924.77	98879.98	15093.83	662523.58	101132.71	670026.99	25
36	14953.53	98875.63	15123.58	661219.19	101137.75	668738.21	24
37	14982.30	98871.28	15153.33	659920.80	101142.60	667454.46	23
38	15011.06	98866.92	15183.09	658627.39	101146.06	666175.68	22
39	15039.81	98862.55	15212.85	657338.92	101150.53	664901.84	21
40	15068.57	98858.17	15242.61	656055.38	101155.02	663632.93	20
41	15097.33	98853.78	15272.38	654776.72	101159.50	661368.90	19
42	15126.08	98849.38	15301.15	653502.93	101164.00	661109.73	18
43	15154.84	98844.98	15331.92	652233.96	101168.51	659855.40	17
44	15183.59	98840.57	15361.89	650969.81	101173.03	658605.87	16
45	15212.34	98836.15	15391.47	649710.43	101177.56	657361.12	15
46	15241.09	98832.72	15421.25	648455.81	101182.09	656121.12	14
47	15269.84	98827.28	15451.03	647205.91	101186.63	654885.86	13
48	15298.58	98822.83	15480.82	645960.70	101191.18	653655.28	12
49	15327.33	98818.38	15510.61	644780.17	101195.74	652429.38	11
50	15356.07	98813.92	15540.40	643484.28	101200.31	651208.12	10
51	15384.82	98809.45	15570.19	642253.01	101204.89	649991.48	9
52	15413.56	98804.97	15599.98	641026.33	101209.48	648779.44	8
53	15442.30	98800.48	15689.78	639804.22	101214.08	647571.97	7
54	15471.04	98795.98	15859.58	638586.65	101218.69	646369.01	6
55	15499.78	98791.48	15869.33	637373.59	101223.31	645170.59	5
56	15528.51	98786.97	15719.19	636165.02	101227.93	643976.66	4
57	15557.25	98782.45	15749.00	634960.92	101232.56	642787.19	3
58	15585.98	98777.91	15779.81	633761.26	101237.80	641602.16	2
59	15614.71	98773.48	15808.61	632566.01	101241.85	640421.54	1
60	15643.45	98768.83	15838.44	631375.15	101246.51	639245.32	0

Canon Sinuum, Tangentium & Secantium.

9	Sinus	Tangens	Secans				
0	15643.41	98768.83	15838.44	631375.15	101246.51	639245.32	60
1	15672.18	98764.28	15868.26	630188.66	101251.18	638073.47	59
2	15700.91	98759.73	15898.08	639006.51	101255.86	636905.95	58
3	15729.63	98755.15	15927.91	627828.68	101260.55	635748.76	57
4	15758.36	98750.57	15957.74	626655.14	101265.25	634583.86	56
5	15787.08	98745.98	15987.57	625485.88	101269.96	633429.23	55
6	15815.81	98741.38	16017.40	624320.86	101274.67	632378.84	54
7	15844.53	98736.77	16047.14	623160.07	101279.39	631138.69	53
8	15873.25	98732.16	16077.08	622003.47	101284.13	629990.73	52
9	15901.97	98727.54	16106.91	610851.06	101288.86	628852.95	51
10	15930.69	98721.91	16136.77	619702.79	101293.61	627719.33	50
11	15959.40	98718.27	16166.62	618558.67	101298.37	626589.84	49
12	15988.12	98713.63	16196.47	617418.65	101303.14	625464.46	48
13	16016.83	98708.97	16226.31	616282.72	101307.92	624343.16	47
14	16045.55	98704.31	16256.17	615150.85	101312.71	623225.94	46
15	16074.26	98699.64	16286.03	614023.03	101317.51	622112.75	45
16	16102.97	98694.96	16315.89	612899.23	101322.31	621003.59	44
17	16131.67	98690.27	16345.76	611779.43	101327.12	619898.43	43
18	16160.38	98685.57	16375.63	610663.60	101331.94	618797.25	42
19	16189.09	98680.86	16405.50	609554.74	101336.77	617700.03	41
20	16217.79	89676.15	16435.37	608443.81	101341.61	616606.74	40
21	16246.50	98671.43	16465.25	607339.79	101346.46	615517.36	39
22	16275.20	98666.70	16495.13	606239.67	101351.31	614431.89	38
23	16303.90	98661.96	16525.01	605143.43	101356.19	613350.28	37
24	16332.60	98657.21	16554.89	604051.03	101361.07	612272.53	36
25	16361.29	98652.46	16584.78	602902.47	101365.95	611198.61	35
26	16389.99	98647.70	16614.67	601877.72	101370.84	610128.50	34
27	16418.68	98642.93	16644.56	600796.76	101375.74	609062.19	33
28	16447.38	98638.15	16674.46	599719.57	101380.65	607099.64	32
29	16476.07	98633.36	16704.36	598646.14	101385.57	606940.85	31
30	16504.76	98628.56	16734.26	597576.44	101390.50	605885.80	30
31	16533.45	98623.75	16764.16	596510.45	101395.44	604814.45	29
32	16562.14	98618.94	16794.07	595448.15	101400.39	603786.80	28
33	16590.83	98614.12	16833.98	594389.52	101405.35	602742.82	27
34	16619.51	98609.29	16833.89	593334.55	101410.32	601702.50	26
35	16648.19	98604.45	16833.81	592283.22	101415.30	600565.81	25
36	16676.87	98599.60	16913.73	591235.50	101420.29	599633.74	24
37	16705.55	98594.74	16943.65	590191.38	101425.29	598603.26	23
38	16734.23	98589.88	16973.58	589150.84	101430.29	597177.37	22
39	16762.91	98585.01	17003.51	588113.86	101435.30	596555.04	21
40	16791.59	98580.13	17033.44	587080.42	101440.32	595536.25	20
41	16820.26	98575.24	17063.37	586050.51	101445.35	594510.98	19
42	16848.94	98570.34	17093.31	585024.10	101450.39	593509.22	18
43	16877.61	98565.44	17123.25	584001.17	101455.44	592500.95	17
44	16906.28	98560.53	17153.19	582981.72	101460.50	591496.14	16
45	16934.95	98555.61	17183.14	581965.72	101465.57	590494.79	15
46	16963.63	98550.68	17213.09	580953.15	101470.64	589496.88	14
47	16992.38	98545.74	17243.04	579944.00	101475.73	588502.38	13
48	17020.99	98540.79	17273.00	578938.25	101480.81	587521.28	12
49	17049.61	98535.83	17303.96	577935.88	101485.91	586533.56	11
50	17078.28	98530.87	17333.91	576936.88	101491.02	585539.20	10
51	17106.94	98525.92	17363.88	575941.82	101496.14	584558.20	9
52	17135.60	98520.92	17393.85	574948.89	101501.27	583530.53	8
53	17164.25	98515.93	17423.82	573919.88	101506.41	582636.17	7
54	17193.91	98510.93	17453.79	572974.16	101511.56	581635.10	6
55	17221.56	98505.92	17483.77	571991.73	101516.71	580667.21	5
56	17250.23	98500.91	17513.75	571012.56	101521.89	579702.80	4
57	17278.87	98495.89	17543.73	570036.63	101527.07	578741.53	3
58	17307.52	98490.86	17573.72	569063.94	101532.26	577783.50	2
59	17335.17	98485.82	17603.71	568094.46	101537.46	576818.67	1
60	17364.82	98480.77	17633.70	567128.18	101542.67	575877.05	0

Canon Sinuum, Tangentium & Secantium.

10	Sinus	Tangens	Secans				
0	17364.82	98480.77	17632.70	567128.18	101542.67	579877.05	60
1	17393.46	98475.71	17662.69	566165.09	101547.88	574928.61	59
2	17422.11	98470.65	17692.69	565205.16	101553.10	573983.33	58
3	17450.73	98465.58	17722.69	564248.38	101558.33	573041.21	57
4	17479.39	98460.50	17752.69	563294.74	101563.57	572102.23	56
5	17508.03	98455.41	17782.70	562344.21	101568.82	571166.36	55
6	17536.67	98450.31	17812.71	561396.80	101574.08	570233.60	54
7	17565.31	98445.21	17842.72	560452.47	101579.35	569303.93	53
8	17593.95	98440.10	17872.74	559511.21	101584.63	568377.34	52
9	17622.58	98434.98	17902.76	558573.04	101589.92	567453.80	51
10	17651.21	98429.83	17932.78	557637.86	101595.18	566453.32	50
11	17679.84	98424.71	17962.81	556705.74	101600.11	565615.84	49
12	17708.47	98419.56	17992.84	555776.63	101605.81	564701.40	48
13	17737.10	98414.40	18022.87	554850.52	101611.14	563789.95	47
14	17765.73	98409.34	18052.91	553927.40	101616.47	562881.48	46
15	17794.35	98404.07	18082.95	553007.34	101621.81	561975.99	45
16	17822.98	98398.89	18112.99	552090.05	101627.16	561073.45	44
17	17851.60	98393.70	18143.03	551175.79	101634.52	560173.86	43
18	17880.22	98388.50	18173.08	550264.46	101637.89	559277.89	42
19	17908.84	98383.29	18203.13	549356.04	101643.27	558383.43	41
20	17937.46	98378.08	18233.18	548450.52	101648.66	557492.78	40
21	17966.07	98372.86	18263.24	547547.88	101654.06	556604.60	39
22	17994.69	98367.63	18293.30	546648.12	101659.46	555529.50	38
23	18023.30	98361.39	18323.36	545751.21	101664.87	554837.26	37
24	18051.91	98357.14	18353.43	544857.15	101670.29	553917.86	36
25	18080.52	98351.89	18383.50	543965.92	101675.78	553081.29	35
26	18109.13	98346.63	18413.57	543077.50	101681.16	552207.54	34
27	18137.74	98341.36	18443.65	542191.88	101686.61	551336.59	33
28	18166.35	98336.08	18473.73	541309.06	101692.07	550468.13	32
29	18194.95	98330.79	18503.81	540429.01	101697.54	549603.05	31
30	18223.55	98325.49	18533.90	539551.72	101703.02	541740.43	30
31	18252.15	98320.18	18563.99	538677.18	101708.51	541780.55	29
32	18280.75	98314.87	18594.08	537805.35	101714.01	547023.42	28
33	18309.35	98309.55	18624.18	536936.30	101719.52	546169.01	27
34	18337.95	98304.22	18654.28	536069.93	101725.04	545317.31	26
35	18366.54	98308.88	18684.38	535206.26	101730.50	544468.31	25
36	18395.13	98293.53	18714.49	534345.27	101736.09	543611.99	24
37	18433.73	98288.17	18744.60	533486.96	101741.01	542778.35	23
38	18452.32	98282.81	18774.71	532631.31	101747.18	541937.37	22
39	18480.91	98277.44	18804.83	531778.30	101752.47	541099.03	21
40	18509.49	98272.06	18834.95	530927.93	101758.01	540263.33	20
41	18538.08	98266.67	18865.07	530080.18	101763.89	539430.26	19
42	18566.66	98261.27	18895.20	529235.05	101769.48	538599.79	18
43	18595.24	98255.87	18925.33	528391.51	101775.08	537771.92	17
44	18623.82	98250.46	18955.46	527552.55	101780.69	536946.64	16
45	18652.40	98245.04	18985.59	526715.17	101786.31	536123.93	15
46	18680.98	98239.61	19015.73	525804.35	101791.94	535303.79	14
47	18709.56	98234.17	19045.87	525048.09	101797.58	534486.20	13
48	18738.13	98228.72	19076.03	524218.36	101803.22	533671.14	12
49	18766.70	98223.27	19106.17	523391.16	101808.87	532858.61	11
50	18795.27	98217.81	19136.32	522556.47	101814.53	532048.60	10
51	18823.84	98212.34	19166.48	521744.28	101820.10	531241.09	9
52	18852.41	98206.86	19196.64	520924.59	101825.88	530436.08	8
53	18880.98	98201.37	19226.80	520107.98	101831.57	529633.54	7
54	18909.54	98195.87	19256.96	519292.64	101837.37	528833.47	6
55	18938.11	98190.36	19287.13	518480.35	101842.98	528035.87	5
56	18966.67	98184.85	19317.30	517670.51	101848.70	527240.70	4
57	18995.13	98179.33	19347.48	516863.11	101854.43	526447.98	3
58	19013.79	98173.80	19377.66	516058.13	101860.17	525657.08	2
59	19051.34	98168.26	19407.84	515255.57	101865.92	524869.79	1
60	19030.90	98163.71	19438.03	514455.40	101871.68	524084.31	0

Canon Sinuum, Tangentium & Secantium.

11	Sinus	Tangens	Secans	
0	19080.90	98162.71	19438.03	514455.40
1	19109.45	98157.16	19468.28	513657.63
2	19138.00	98151.60	19498.41	512862.24
3	19166.55	98146.03	19528.61	512069.21
4	19195.10	98140.45	19558.81	511278.55
5	19223.65	98134.86	19589.01	510490.24
6	19252.30	98129.26	19619.21	509704.26
7	19280.74	98123.66	19649.43	508920.61
8	19309.28	98118.05	19679.64	508139.28
9	19337.82	98114.43	19709.86	507360.25
10	19366.36	98108.80	19740.08	506583.52
11	19394.90	98104.16	19770.30	505809.07
12	19423.44	98095.51	19800.53	505036.90
13	19451.97	98089.86	19830.76	504867.00
14	19480.50	98084.20	19861.00	503499.37
15	19509.03	98078.18	19891.24	502738.97
16	19537.56	98072.85	19921.48	501970.78
17	19566.09	98067.16	19951.72	501209.84
18	19594.61	98061.46	19981.97	500451.11
19	19623.14	98055.76	20011.22	499694.79
20	19651.66	98050.05	20041.48	498940.27
21	19680.18	98044.33	20071.74	498188.13
22	19708.70	98038.60	20103.00	497438.17
23	19737.22	98032.86	20133.27	496690.37
24	19765.73	98027.11	20163.54	495944.74
25	19794.25	98021.36	20193.82	495201.25
26	19822.76	98015.60	20224.09	494459.90
27	19851.27	98009.83	20254.37	493720.68
28	19879.78	98004.05	20284.65	492983.58
29	19908.29	97998.26	20314.94	492248.59
30	19936.79	97993.47	20345.23	491515.70
31	19965.30	97986.67	20375.52	490784.91
32	19993.80	97980.86	20405.81	490056.25
33	20022.30	97975.04	20436.12	489329.76
34	20050.80	97969.21	20466.43	488604.99
35	20079.30	97963.37	20496.74	487831.48
36	20107.79	97957.53	20527.05	487162.01
37	20136.29	97951.67	20557.37	486443.59
38	20164.78	97945.81	20587.69	485727.89
39	20193.27	97939.94	20614.01	485013.82
40	20221.76	97934.05	20648.34	484300.45
41	20250.24	97928.17	20678.67	483590.10
42	20278.73	97922.28	20709.00	482881.74
43	20307.21	97916.38	20739.34	482175.36
44	20335.69	97910.47	20769.68	481470.96
45	20364.17	97904.55	20800.03	480768.54
46	20392.65	97898.62	20830.38	480068.08
47	20421.13	97892.68	20860.73	479369.57
48	20449.61	97886.74	20891.09	478673.00
49	20478.08	97880.79	20921.45	477998.37
50	20506.55	97874.83	20951.81	477285.67
51	20535.02	97868.86	20981.18	476594.90
52	20563.49	97862.88	21012.55	475906.03
53	20591.95	97856.89	21043.93	475219.07
54	20620.42	97850.90	21073.31	474534.01
55	20648.38	97844.90	21103.69	473850.85
56	20677.34	97838.89	21134.07	473169.54
57	20705.80	97832.87	21164.46	472490.11
58	20734.26	97826.84	21194.85	471811.96
59	20763.71	97820.80	21225.23	471136.86
60	20791.17	97814.76	21255.65	470453.01

Canon Sinuum, Tangentium & Secantium.

12	Sinus	Tangens	Secans				
0	20791.17	97814.76	21255.65	470463.01	102234.07	480971.43	60
1	20819.62	97808.71	21286.06	469791.00	102240.40	480316.13	59
2	20848.07	97802.65	21316.47	469120.83	102246.73	479660.66	58
3	20876.52	97796.58	21346.88	468452.48	102253.07	479007.02	57
4	20904.97	97790.50	21377.30	467785.95	102259.42	478355.20	56
5	20933.41	97784.41	21407.72	467121.24	102265.78	477705.19	55
6	20961.86	97778.32	21438.14	466458.38	102272.15	477056.99	54
7	20990.30	97772.82	21468.57	465797.21	102278.53	476410.58	53
8	21018.74	97766.11	21499.00	465137.88	102284.92	475745.96	52
9	21047.18	97759.99	21529.44	464480.34	102291.32	475123.12	51
10	21075.61	97753.86	21559.88	463824.57	102297.73	474483.06	50
11	21104.05	97747.73	21590.32	463170.56	102304.15	473843.77	49
12	21132.48	97741.59	21620.77	462518.32	102310.58	473205.23	48
13	21160.91	97735.44	21651.23	461867.83	102317.02	472569.45	47
14	21189.34	97729.28	21681.67	461219.08	102323.47	471935.42	46
15	21217.77	97723.11	21712.13	460572.07	102329.93	471303.13	45
16	21246.19	97716.93	21742.59	459926.80	102336.40	470671.56	44
17	21274.62	97710.75	21773.06	459183.35	102342.88	470043.72	43
18	21303.04	97704.56	21803.53	458641.41	102349.37	469416.60	42
19	21331.46	97698.36	21834.00	458001.29	102355.87	468791.19	41
20	21359.88	97692.15	21864.48	457362.87	102362.38	468167.48	40
21	21388.29	97685.93	21894.96	456726.14	102368.90	467545.48	39
22	21416.71	97679.70	21925.44	456091.55	102375.43	466981.16	38
23	21445.12	97673.47	21955.93	455477.76	102381.96	466306.52	37
24	21473.53	97667.23	21986.42	454826.08	102388.50	465689.56	36
25	21501.94	97660.98	22016.92	454196.08	102395.05	465074.27	35
26	21530.35	97654.73	22047.41	453567.73	102401.61	464460.64	34
27	21558.76	97648.45	22077.93	452941.05	102408.18	463848.67	33
28	21587.16	97642.17	22108.44	452316.01	102414.76	463238.33	32
29	21615.56	97635.89	22138.95	451692.61	102421.35	462629.67	31
30	21643.96	97629.60	22169.47	451070.85	102427.95	462022.63	30
31	21671.36	97623.30	22199.99	450450.72	102434.56	461417.22	29
32	21700.76	97616.99	22230.51	449832.21	102441.18	460813.43	28
33	21729.15	97610.67	22261.04	449215.32	102447.81	460111.26	27
34	21757.54	97604.35	22291.57	448600.04	102454.45	459610.70	26
35	21785.93	97598.02	22322.11	447986.36	102461.10	459011.74	25
36	21814.32	97591.68	22352.65	447374.28	102467.76	458414.39	24
37	21842.71	97585.33	22383.19	446763.79	102474.43	457818.61	23
38	21871.10	97578.97	22413.74	446154.89	102481.11	457224.44	22
39	21899.48	97572.60	22444.29	445547.56	102487.80	456331.83	21
40	21927.86	97566.23	22474.85	444941.81	102494.49	456040.80	20
41	21956.24	97559.85	22505.41	444337.62	102501.19	455451.34	19
42	21984.62	97553.46	22535.97	443734.99	102507.90	454863.44	18
43	22013.00	97547.06	22566.54	443133.92	102514.62	454277.09	17
44	22041.37	97540.65	22597.11	442534.39	102521.35	453692.29	16
45	22069.74	97534.23	22627.69	441936.41	102528.09	453109.03	15
46	22098.11	97527.81	22658.27	441339.96	102534.84	452527.30	14
47	22126.48	97521.38	22688.85	440745.04	102541.60	451947.11	13
48	22154.85	97514.94	22719.44	440151.64	102548.37	451368.44	12
49	22183.21	97508.49	22750.03	439559.76	102555.15	450791.29	11
50	22211.58	97502.03	22780.63	438969.40	102561.94	450215.65	10
51	22239.94	97495.56	22811.23	438380.54	102568.74	449641.52	9
52	22266.30	97489.09	22842.83	437793.17	102575.55	449068.89	8
53	22296.66	97483.61	22873.44	437207.31	102582.37	448497.75	7
54	22325.01	97476.12	22903.05	436622.93	102589.20	447928.10	6
55	22353.37	97469.62	22933.67	436040.03	102596.04	447359.93	5
56	22381.72	97463.11	22964.29	435458.61	102602.89	446793.24	4
57	22410.07	97456.60	22994.92	434878.66	102609.75	446638.03	3
58	22438.41	97450.08	23025.55	434300.18	102616.62	445664.28	2
59	22466.76	97443.55	23056.18	433728.16	102623.50	445101.98	1
60	22495.11	97437.01	23086.82	433147.59	102630.39	444541.15	0

Canon Sinuum, Tangentium & Secantium.

13	Sinus	Tangens	Secans	
0	23495.11	97437.08	23086.83	433347.59
1	23523.45	97430.46	23117.46	432573.47
2	23551.79	97433.90	23148.11	433000.79
3	23580.13	97417.34	23178.76	431429.55
4	23608.46	97410.77	23209.41	430559.74
5	23636.80	97404.19	23240.07	430391.36
6	23665.13	97397.60	23270.73	429734.40
7	23693.46	97391.00	23301.40	429158.85
8	23721.79	97384.39	23332.07	428594.72
9	23750.13	97377.78	23362.74	428031.99
10	23778.44	97371.16	23393.43	427470.66
11	23806.77	97364.53	23424.10	426910.72
12	23835.09	97357.89	23454.79	426352.18
13	23863.41	97351.24	23485.48	425795.01
14	23891.73	97344.58	23516.17	425139.23
15	23920.04	97337.92	23546.87	424684.83
16	23948.35	97331.25	23577.58	424131.77
17	23976.66	97324.57	23608.29	423580.09
18	23004.97	97317.88	23639.00	423029.77
19	23033.28	97311.18	23669.72	422480.80
20	23061.59	97304.48	23700.44	421933.18
21	23089.89	97297.77	23731.16	421386.90
22	23118.19	97291.09	23761.89	420841.96
23	23146.49	97284.32	23792.62	420398.35
24	23174.79	97277.58	23823.36	419756.06
25	23203.09	97270.84	23854.10	419115.10
26	23231.38	97264.09	23884.85	418675.46
27	23259.67	97257.33	23915.60	418191.13
28	23287.96	97250.56	23946.35	417600.21
29	23316.25	97243.78	23977.11	417064.40
30	23344.54	97236.99	24007.87	416539.98
31	23372.82	97230.19	24038.64	415996.85
32	23401.10	97223.39	24069.41	415465.01
33	23429.38	97216.58	24100.19	414934.46
34	23457.66	97209.76	24130.97	414405.19
35	23485.94	97202.93	24161.76	413877.19
36	23514.21	97196.09	24192.55	413330.46
37	23542.48	97189.25	24223.34	412544.99
38	23570.75	97182.40	24254.14	412300.79
39	23599.02	97175.54	24284.94	411797.84
40	23627.29	97168.67	24315.75	411156.84
41	23655.55	97161.79	24346.56	410735.69
42	23683.81	97154.91	24376.37	410316.49
43	23711.07	97148.02	24408.19	409668.51
44	23740.33	97141.12	24439.01	409181.78
45	23768.59	97134.21	24469.84	408666.27
46	23796.84	97127.39	24500.67	408151.99
47	23825.10	97120.56	24531.51	407638.93
48	23853.35	97113.43	24562.35	407127.07
49	23881.59	97106.49	24593.20	406616.43
50	23909.84	97099.54	24624.05	406107.00
51	23938.08	97092.58	24654.91	405598.77
52	23966.33	97085.61	24685.77	405091.74
53	23994.17	97078.61	24716.63	404585.90
54	24022.80	97071.65	24747.50	404081.25
55	24051.04	97064.66	24778.37	403577.79
56	24079.27	97057.66	24809.25	403075.50
57	24107.51	97050.65	24840.13	402574.40
58	24135.74	97043.63	24871.02	402074.46
59	24163.96	97036.66	24901.91	401575.70
60	24192.19	97029.57	24932.80	401078.09

Canon Sinuum, Tangentium & Secantium.

14	Sinus	Tangens	Secans				
01	24392.19	97089.57	24932.80	401078.09	103061.35	413356.55	60
1	24320.41	97012.53	24963.70	400381.63	103068.83	412874.87	59
2	24248.63	97015.48	24994.60	400086.36	103076.32	413394.35	58
3	24176.85	970084.42	25025.51	399591.23	103083.81	411944.98	57
4	24305.07	97001.35	25056.42	399099.24	103091.33	411436.75	56
5	24333.29	96994.28	25087.34	398607.39	103098.85	410959.67	55
6	24361.50	96987.20	25118.26	398116.69	103106.38	410483.74	54
7	24389.71	96980.11	25149.19	397637.13	103113.91	410008.93	53
8	24417.92	96973.01	25180.12	397138.68	103121.47	409735.16	52
9	24446.13	96965.90	25211.06	396661.37	103129.03	409062.72	51
10	24474.33	96958.79	25242.00	396161.18	103136.60	408591.30	50
11	24501.54	96951.67	25272.94	395680.11	103144.18	408121.00	49
12	24530.74	96944.54	25303.89	395196.15	103151.77	407651.81	48
13	24558.94	96937.40	25334.84	394713.31	103159.36	407183.74	47
14	24587.13	96930.29	25365.80	394331.57	103166.97	406716.77	46
15	24615.33	96923.09	25396.76	393750.94	103174.59	406130.91	45
16	24643.52	96915.93	25417.73	393274.41	103182.23	405756.15	44
17	24671.71	96908.75	25448.70	392792.97	103189.85	405322.49	43
18	24699.90	96901.57	25479.68	392315.63	103197.50	404859.93	42
19	24728.09	96894.38	25510.66	391839.37	103205.16	404398.44	41
20	24756.27	96887.18	25531.65	391364.20	103212.82	403938.04	40
21	24784.45	96879.98	25562.64	390890.11	103220.50	403478.73	39
22	24812.63	96872.77	25613.63	390417.10	103228.18	403010.48	38
23	24840.81	96865.55	25644.63	389945.16	103235.88	402563.32	37
24	24868.99	96858.32	25675.63	389474.29	103243.59	402107.22	36
25	24897.16	96851.08	25706.64	388004.48	103251.30	401652.19	35
26	24925.33	96843.83	25737.66	388535.74	103259.03	401198.23	34
27	24953.50	96836.57	25768.68	388068.05	103266.76	400745.33	33
28	24981.67	96829.31	25799.70	387601.41	103274.51	400293.47	32
29	25009.84	96822.04	25830.73	387135.84	103282.27	399842.67	31
30	25038.00	96814.76	25861.76	386671.31	103290.03	399392.93	30
31	25066.16	96807.47	25892.80	386107.88	103297.81	398944.21	29
32	25094.32	96800.18	25923.84	385745.37	103305.59	398496.54	28
33	25122.48	96792.88	25954.88	385218.36	103313.39	398049.91	27
34	25150.63	96785.57	25985.93	384813.58	103321.19	397604.31	26
35	25178.79	96778.35	26016.99	384364.24	103329.01	397159.75	25
36	25206.94	96770.92	26048.05	383905.91	103336.83	396716.21	24
37	25235.08	96763.58	26079.11	383448.61	103344.67	396373.69	23
38	25263.23	96756.23	26110.18	381992.33	103352.51	395832.19	22
39	25291.37	96748.88	26141.26	381532.07	103360.37	395391.71	21
40	25319.52	96741.52	26172.34	381024.81	103368.23	394952.24	20
41	25347.66	96734.15	26203.42	380629.57	103376.11	394513.79	19
42	25371.79	96726.77	26234.51	381177.33	103383.99	394076.33	18
43	25403.93	96719.38	26265.60	380726.09	103391.88	393639.88	17
44	25431.06	96711.99	26296.70	380275.85	103399.79	393204.43	16
45	25460.19	96704.59	26327.80	379836.61	103407.70	392769.97	15
46	25488.33	96697.18	26358.93	379378.35	103415.63	392336.51	14
47	25516.45	96690.76	26390.02	378931.04	103423.50	391904.03	13
48	25544.58	96682.33	26421.14	378484.81	103431.51	391472.54	12
49	25572.70	96674.90	26452.26	378039.51	103439.46	391042.03	11
50	25600.82	96667.46	26483.39	377597.19	103447.43	390612.90	10
51	25628.94	96660.01	26514.52	377153.85	103455.40	390183.95	9
52	25657.09	96652.51	26545.66	376709.47	103463.38	389756.37	8
53	25685.17	96645.08	26576.80	376268.07	103471.38	389339.76	7
54	25713.28	96637.60	26607.94	375837.63	103479.38	388904.11	6
55	25741.39	96630.18	26639.09	375388.15	103487.40	388479.43	5
56	25769.50	96622.63	26670.25	374949.63	103495.42	388055.70	4
57	25797.60	96615.13	26701.41	374432.07	103503.46	387632.93	3
58	25825.70	96607.68	26732.57	374071.46	103511.50	387211.12	2
59	25853.81	96600.10	26763.74	373639.80	103519.55	386790.23	1
60	25881.90	96592.58	26794.92	373205.08	103527.62	386370.33	0

Canon Sinuum, Tangentium & Secantium.

15	Sinus	Tangens	Secans	
0	25881.90	96593.58	26794.91	373203.08
1	25910.00	96585.05	26826.10	371771.31
2	25938.10	96577.51	26857.23	371338.47
3	25966.19	96569.96	26888.47	371065.58
4	25994.28	96561.40	26919.67	371475.61
5	26022.37	96553.81	26950.87	371045.58
6	26050.45	96547.26	26981.07	370616.48
7	26078.53	96539.68	27013.28	370188.30
8	26106.61	96532.09	27044.49	369761.03
9	26134.69	96524.49	27075.71	369334.69
10	26162.77	96516.88	27106.93	368909.27
11	26190.85	96509.27	27138.16	368484.75
12	26218.93	96501.65	27169.40	368061.15
13	26246.99	96494.02	27200.64	367638.45
14	26275.06	96486.38	27231.88	367215.65
15	26303.12	96478.73	27263.13	366795.75
16	26331.18	96471.07	27294.38	366375.75
17	26359.24	96463.41	27325.64	365956.65
18	26387.30	96455.74	27356.90	365538.44
19	26415.36	96448.06	27388.17	365121.11
20	26443.42	96440.37	27419.44	364704.67
21	26471.47	96432.67	27450.72	364289.11
22	26499.53	96424.97	27482.01	363874.44
23	26527.57	96417.26	27513.30	363462.64
24	26555.61	96409.54	27544.59	363047.71
25	26583.65	96401.81	27575.89	362635.66
26	26611.69	96394.07	27607.19	362224.47
27	26639.73	96386.33	27638.50	361814.15
28	26667.77	96378.58	27669.81	361404.69
29	26695.81	96370.82	27701.13	360996.09
30	26723.84	96363.05	27732.45	360588.35
31	26751.87	96355.27	27763.78	360181.45
32	26779.89	96347.48	27795.12	359775.43
33	26807.92	96339.69	27826.46	359370.34
34	26835.94	96331.80	27857.80	358965.90
35	26863.96	96324.08	27889.15	358562.41
36	26891.98	96316.26	27920.50	358159.75
37	26919.00	96308.43	27951.86	357777.94
38	26948.02	96300.59	27983.22	357356.96
39	26976.02	96292.75	28014.59	356956.81
40	27004.03	96284.90	28045.97	356557.49
41	27032.04	96277.04	28077.35	356159.00
42	27060.04	96269.17	28108.73	355761.33
43	27088.05	96261.30	28140.12	355364.49
44	27116.05	96253.42	28171.52	354966.46
45	27144.04	96245.53	28202.92	354573.25
46	27172.04	96237.63	28234.32	354178.86
47	27200.03	96229.71	28265.73	353783.28
48	27228.02	96221.80	28297.15	353392.51
49	27256.01	96213.87	28328.57	353000.54
50	27284.00	96205.94	28359.99	352609.38
51	27311.98	96198.00	28391.42	352219.02
52	27339.96	96190.05	28422.86	351829.46
53	27367.94	96182.09	28454.30	351440.70
54	27395.92	96174.13	28485.73	351052.73
55	27423.90	96166.16	28517.20	350665.55
56	27451.87	96158.18	28548.66	350279.16
57	27479.84	96150.19	28580.12	349893.56
58	27507.81	96142.19	28611.59	349508.74
59	27535.78	96134.18	28643.06	349124.70
60	27563.74	96126.17	28674.54	348741.44

Canon Sinuum, Tangentium & Secantium .

16	Sinus	Tangens	Secans	
0	27563.74	96126.17	28674.54	348741.44
1	27591.70	96118.15	28706.02	34835.96
2	27619.65	96110.12	28737.51	347977.26
3	27647.61	96102.08	28769.00	347596.32
4	27675.56	96094.03	28800.50	347216.16
5	27703.52	96085.98	28832.01	346836.76
6	27731.47	96077.92	28863.52	346458.13
7	27759.41	96069.85	28895.03	346080.26
8	27787.36	96061.77	28926.55	345703.15
9	27815.30	96053.68	28958.08	345326.79
10	27843.24	96045.58	28989.61	344971.20
11	27871.18	96037.48	29021.14	344576.35
12	27899.11	96029.37	29052.68	344202.26
13	27927.04	96021.25	29084.23	343828.91
14	27954.97	96013.12	29115.78	343456.31
15	27982.90	96004.98	29147.34	343084.46
16	28010.83	95996.84	29178.90	342713.34
17	28038.75	95988.69	29210.47	342342.97
18	28066.67	95980.53	29242.05	341973.33
19	28094.59	95972.36	29273.63	341604.41
20	28122.51	95964.18	29305.21	341236.26
21	28150.42	95956.00	29336.80	340868.82
22	28178.33	95947.81	29368.39	340502.10
23	28206.24	95939.61	29399.99	340136.12
24	28234.15	95931.40	29431.60	339970.85
25	28262.05	95923.18	29463.21	339406.31
26	28289.95	95914.95	29494.83	339042.49
27	28317.85	95906.72	29526.45	338679.38
28	28345.75	95898.48	29558.08	338316.99
29	28373.64	95890.28	29589.71	337955.31
30	28401.53	95881.97	29621.35	337594.34
31	28429.42	95873.70	29652.99	337234.08
32	28457.31	95865.43	29684.64	336874.53
33	28485.20	95857.15	29716.30	336515.68
34	28513.08	95848.86	29747.96	336157.53
35	28540.96	95840.56	29779.62	335800.08
36	28568.84	95832.25	29811.29	335443.33
37	28596.71	95823.94	29842.97	335087.38
38	28624.58	95815.62	29874.65	334711.91
39	28652.45	95807.29	29906.34	334377.24
40	28680.32	95798.95	29938.03	334023.26
41	28708.19	95790.65	29969.73	333669.97
42	28736.05	95782.35	30001.44	333317.36
43	28763.91	95773.89	30033.15	332965.43
44	28791.77	95765.52	30064.86	332614.19
45	28819.63	95757.14	30096.58	332263.62
46	28847.48	95748.75	30128.31	331913.73
47	28875.33	95740.35	30160.04	331564.52
48	28903.18	95731.95	30191.78	331215.98
49	28931.03	95723.54	30223.52	330868.11
50	28958.87	95715.12	30255.27	330580.91
51	28986.71	95706.69	30287.03	330174.38
52	29014.55	95698.25	30318.79	329838.51
53	29041.39	95689.81	30350.55	329483.30
54	29070.22	95681.36	30382.32	329138.76
55	29098.05	95672.90	30414.10	328794.87
56	29125.88	95664.43	30445.88	328451.64
57	29153.71	95655.95	30477.67	328109.07
58	29181.53	95647.47	30509.46	327767.15
59	29209.35	95638.98	30541.26	327425.88
60	29237.17	95630.48	30573.07	327085.86

Canon Sinuum, Tangentium & Secantium.

17	Sinus	Tangens	Secans	
0	19237. 17	95630.48	30573.07	327085.26
1	19164.99	95611.97	30604.88	326745.29
2	29192.80	95613.45	30636.69	326405.96
3	29320.61	95604.92	30668.51	326067.28
4	29348.43	95596.39	30700.34	325739.24
5	29376.23	95587.85	30733.18	325391.84
6	29404.03	95579.30	30764.02	325057.58
7	29431.83	95570.74	30795.86	324718.95
8	29459.63	95562.17	30827.71	324383.46
9	29487.43	95553.60	30859.57	324048.60
10	29515.23	95545.02	30891.43	323714.38
11	29543.01	95536.43	30923.30	323380.78
12	29570.80	95527.83	30955.37	323047.80
13	29598.59	95519.22	30987.05	322715.46
14	29626.38	95510.61	31018.93	322383.73
15	29654.16	95501.99	31050.81	322053.63
16	29681.94	95493.36	31082.72	321783.15
17	29709.71	95484.72	31114.62	321393.28
18	29737.49	95476.07	31146.53	321063.04
19	29765.26	95467.42	31178.44	320734.40
20	29793.03	95458.76	31210.36	320406.38
21	29820.79	95450.09	31242.29	320078.97
22	29848.56	95441.41	31274.22	319752.17
23	29876.33	95433.72	31306.16	319445.98
24	29904.08	95424.03	31338.10	319100.39
25	29931.84	95415.33	31370.05	318775.40
26	29959.59	95406.62	31402.00	318451.02
27	29987.34	95397.90	31433.90	318137.84
28	30015.09	95389.17	31465.93	317804.06
29	30042.84	95380.43	31497.90	317481.47
30	30070.58	95371.69	31529.88	317159.48
31	30098.31	95362.94	31561.86	316838.08
32	30126.06	95354.18	31593.85	316517.28
33	30153.80	95345.41	31625.85	316197.06
34	30181.53	95336.64	31657.85	315877.44
35	30209.26	95327.86	31689.86	315558.40
36	30236.99	95319.07	31721.87	315239.94
37	30264.71	95310.27	31753.89	314923.07
38	30292.44	95301.46	31785.91	314604.78
39	30320.16	95292.64	31817.94	314288.07
40	30347.88	95283.88	31849.98	313971.94
41	30375.59	95274.99	31882.02	313656.39
42	30403.31	95266.15	31914.07	313341.41
43	30431.02	95257.30	31946.13	313027.01
44	30458.72	95248.44	31978.19	312713.17
45	30486.43	95239.58	32010.25	312399.91
46	30514.13	95230.71	32042.32	312087.22
47	30541.84	95228.83	32074.40	311775.09
48	30569.53	95212.94	32106.49	311463.53
49	30597.23	95204.04	32138.58	311153.54
50	30634.92	95195.14	32170.67	310843.20
51	30662.61	95186.23	32202.77	310533.23
52	30680.29	95177.31	32234.88	310288.91
53	30707.98	95168.38	32267.00	309914.16
54	30735.66	95159.44	32299.12	309605.96
55	30763.34	95150.49	32331.25	309298.31
56	30791.03	95141.54	32363.38	308991.22
57	30818.69	95132.58	32395.52	308684.68
58	30846.36	95123.61	32427.66	308378.69
59	30874.03	95114.63	32459.81	308093.25
60	30901.70	95105.65	32491.97	307768.35

Canon Sinuum, Tangentium & Secantium.

18	Sinus	Tangens	Secans				
0	30901.70	95105.65	32491.97	307768.35	105146.28	323606.80	60
1	30929.36	95096.66	32524.13	307464.00	105156.17	323317.36	59
2	30957.02	95087.66	32556.30	307160.20	105166.12	323018.46	58
3	30984.68	95078.65	32588.48	306856.93	105176.08	322740.11	57
4	31012.34	95069.63	32620.66	306554.21	105186.06	322452.30	56
5	31039.99	95060.60	32652.85	306252.03	105196.05	322165.03	55
6	31067.64	95051.57	32685.04	305950.38	105206.04	321878.30	54
7	31095.39	95042.53	32717.24	305649.28	105216.03	321592.10	53
8	31123.94	95033.48	32749.44	305348.70	105226.07	321306.44	52
9	31150.58	95024.42	32781.63	305048.66	105236.10	321014.32	51
10	31178.22	95015.36	32813.87	304749.15	105246.14	320736.73	50
11	31205.86	95006.29	32846.10	304450.18	105256.19	320452.66	49
12	31233.49	94997.81	32878.33	304151.73	105266.25	320169.13	48
13	31261.12	94988.12	32910.56	303853.39	105276.33	319886.13	47
14	31288.75	94979.02	32942.80	303556.41	105286.41	319603.65	46
15	31316.38	94969.91	32975.05	303259.54	105296.51	319321.70	45
16	31344.00	94960.80	33007.31	302963.20	105306.61	319040.18	44
17	31371.63	94951.68	33039.57	302667.37	105316.73	318759.37	43
18	31399.25	94942.55	33071.84	302374.07	105326.86	318478.99	42
19	31426.86	94933.41	33104.11	302077.28	105336.99	318199.13	41
20	31454.48	94924.26	33136.39	301783.01	105347.14	317919.78	40
21	31482.09	94915.11	33168.68	301489.26	105357.30	317640.95	39
22	31509.69	94905.97	33200.97	301196.02	105367.47	317362.64	38
23	31537.30	94896.78	33233.27	300903.30	105377.65	317084.84	37
24	31564.90	94887.60	33265.57	300611.09	105387.83	316807.56	36
25	31592.50	94878.41	33297.88	300319.39	105398.05	316530.78	35
26	31620.10	94869.22	33330.20	300018.20	105408.26	316254.52	34
27	31647.70	94860.02	33362.52	299737.51	105418.49	315978.76	33
28	31675.39	94850.81	33394.85	299447.34	105428.73	315703.51	32
29	31703.88	94841.59	33427.19	299157.66	105438.97	315418.77	31
30	31730.47	94832.36	33459.53	298868.50	105449.83	315154.53	30
31	31758.05	94823.13	33491.88	298579.83	105459.90	314880.79	29
32	31785.63	94813.89	33524.24	298291.66	105469.78	314607.56	28
33	31813.21	94804.64	33556.60	298004.00	105480.07	314334.83	27
34	31840.79	94795.38	33587.99	297716.83	105490.37	314062.59	26
35	31868.36	94786.11	33621.34	297430.16	105500.68	313790.86	25
36	31895.93	94776.84	33653.72	297143.99	105511.01	313519.63	24
37	31933.50	94767.56	33686.11	296856.31	105521.34	313248.87	23
38	31951.06	94758.27	33718.50	296573.12	105531.69	312978.62	22
39	31978.63	94748.97	33750.90	296288.48	105542.04	312708.86	21
40	32006.19	94739.66	33783.30	296004.22	105552.41	312439.59	20
41	32033.74	94730.35	33815.71	295780.50	105562.79	312170.81	19
42	32061.30	94721.03	33848.13	295437.27	105573.18	311902.52	18
43	32088.85	94711.70	33880.56	295154.53	105583.18	311634.72	17
44	32116.40	94702.36	33911.99	294872.27	105593.99	311367.40	16
45	32143.95	94693.01	33945.43	294590.50	105604.41	311100.57	15
46	32171.49	94683.66	33977.87	294309.21	105614.85	310834.22	14
47	32199.03	94674.30	34010.32	294028.40	105625.29	310568.95	13
48	32226.57	94664.93	34042.78	293748.07	105635.75	310308.96	12
49	32254.10	94655.55	34075.24	293468.23	105646.11	310038.09	11
50	32281.64	94646.16	34107.71	293188.83	105656.69	309773.63	10
51	32309.17	94636.76	34140.19	292909.95	105667.18	309509.67	9
52	32336.70	94627.36	34172.67	292631.53	105677.68	309246.80	8
53	32364.22	94617.95	34205.16	292353.98	105688.19	308963.19	7
54	32391.74	94608.53	34237.65	292076.10	105698.71	308720.66	6
55	32419.26	94599.10	34270.15	291799.09	105709.24	308458.60	5
56	32446.78	94589.67	34302.66	291523.56	105719.78	308197.08	4
57	32474.39	94580.23	34334.18	291246.49	105730.34	307935.90	3
58	32501.80	94570.78	34367.70	290970.89	105740.90	307675.85	2
59	32529.31	94561.32	34400.23	290695.76	105751.48	307415.07	1
60	32556.82	94551.85	34432.76	290421.09	105762.07	307155.35	0

Canon Sinuum, Tangentium & Secantium.

19	Sinus	Tangens	Secans	
0	32556.82	94551.85	34432.76	290421.09
1	32584.32	94542.38	34465.30	290146.88
2	32611.82	94532.90	34497.85	289873.14
3	32639.31	94523.41	34530.40	289199.86
4	32666.81	94513.91	34562.56	289327.04
5	32694.30	94504.40	34595.53	289554.67
6	32721.79	94494.89	34618.10	288782.77
7	32749.28	94485.37	34640.68	288511.32
8	32776.76	94475.84	34693.27	288240.33
9	32804.24	94466.30	34725.86	287969.79
10	32831.72	94456.75	34758.46	287699.70
11	32859.19	94447.20	34791.07	287430.07
12	32886.66	94437.64	34823.68	287160.88
13	32914.13	94428.07	34856.30	286892.15
14	32941.60	94418.49	34888.93	286623.86
15	32969.06	94408.90	34921.56	286356.01
16	32996.52	94399.31	34954.20	286088.63
17	33023.98	94389.71	34986.85	285821.93
18	33051.44	94380.10	35019.50	285555.17
19	33078.89	94370.48	35052.16	285289.11
20	33106.34	94360.85	35084.83	285023.49
21	33133.79	94351.21	35117.50	284758.31
22	33161.23	94341.57	35150.18	284493.56
23	33188.67	94331.92	35181.87	284229.26
24	33216.11	94322.26	35215.56	283965.39
25	33243.55	94312.60	35248.26	283701.96
26	33270.98	94303.93	35280.97	283438.96
27	33298.41	94293.25	35313.68	283176.39
28	33325.84	94283.56	35346.40	282914.26
29	33353.27	94273.86	35379.13	282652.56
30	33380.69	94264.15	35411.86	282391.29
31	33408.10	94254.43	35444.60	282130.45
32	33445.52	94244.71	35477.35	281870.03
33	33483.93	94234.98	35510.10	281616.04
34	33490.34	94225.24	35542.86	281350.48
35	33517.71	94215.50	35575.63	281091.34
36	33545.16	94205.75	35608.40	280832.63
37	33572.56	94197.99	35641.18	280574.33
38	33599.96	94186.23	35673.97	280316.46
39	33627.35	94176.44	35706.76	280059.01
40	33654.75	94166.65	35739.56	279801.98
41	33682.14	94156.85	35772.37	279545.37
42	33709.13	94147.05	35805.18	279289.17
43	33736.91	94137.24	35838.00	279033.39
44	33764.29	94127.42	35870.83	278778.02
45	33791.67	94117.60	35903.67	278513.07
46	33819.05	94107.77	35936.51	278268.53
47	33846.42	94097.93	35969.36	278014.40
48	33873.79	94088.08	36002.22	277770.69
49	33901.16	94078.22	36035.08	277507.38
50	33928.53	94068.35	36067.95	277254.48
51	33955.89	94058.48	36100.83	277001.99
52	33983.25	94048.60	36133.71	276749.90
53	34010.60	94038.71	36166.60	276498.23
54	34037.95	94028.81	36199.50	276246.95
55	34065.30	94018.90	36232.40	275996.08
56	34092.65	94008.99	36265.31	275745.61
57	34120.00	93999.07	36308.23	275495.54
58	34147.34	93989.14	36331.15	275245.83
59	34174.68	93979.20	36364.08	274996.61
60	34202.02	93969.26	36397.01	274747.74

Canon Sinuum, Tangentium & Secantium.

20	Sinus	Tangens	Secans				
0	34203.02	93969.26	36399.02	274747.74	106417.78	292380.44	60
1	34229.35	93959.31	36489.97	274499.27	106429.05	292146.97	59
2	34256.68	93949.35	36462.92	274251.20	106440.33	291913.89	58
3	34284.01	93939.38	36495.88	274003.52	106451.63	291681.21	57
4	34311.33	93929.40	36526.85	273756.23	106462.94	291448.93	56
5	34338.65	93919.42	36561.82	273509.34	106474.26	291217.03	55
6	34365.97	93909.43	36594.80	273262.84	106485.59	290985.53	54
7	34393.29	93899.43	36627.79	273016.74	106496.93	290754.43	53
8	34420.60	93889.42	36660.79	272771.02	106508.28	290528.72	52
9	34447.91	93879.40	36693.79	272525.69	106519.64	290298.39	51
10	34475.22	93869.37	36726.80	272280.75	106531.01	290063.46	50
11	34502.52	93859.34	36759.82	272036.20	106543.40	289833.91	49
12	34529.83	93849.30	36792.84	271793.04	106553.80	289604.75	48
13	34557.12	93839.25	36815.87	271548.26	106565.21	289375.98	47
14	34584.42	93829.19	36838.91	271304.87	106576.63	289147.60	46
15	34611.71	93819.18	36861.95	271061.16	106588.07	288919.59	45
16	34639.00	93809.06	36893.00	270819.23	106599.51	288691.98	44
17	34666.29	93798.98	36915.08	270726.99	106610.97	288464.74	43
18	34693.57	93788.89	36991.13	270335.13	106622.43	288137.89	42
19	34720.85	93778.79	37024.20	270093.04	106633.91	288011.42	41
20	34748.13	93768.69	37057.28	269852.34	106645.40	287785.32	40
21	34775.40	93758.58	37090.37	269611.81	106656.90	287539.61	39
22	34802.67	93748.46	37123.46	269371.47	106668.42	287334.28	38
23	34829.94	93738.33	37156.56	269131.49	106679.94	287109.32	37
24	34857.21	93728.19	37189.67	268891.90	106691.48	286884.74	36
25	34884.47	93718.05	37222.78	268652.67	106703.02	286660.93	35
26	34911.73	93707.90	37255.90	268413.83	106714.58	286436.70	34
27	34938.99	93697.74	37289.03	268175.85	106726.15	286213.24	33
28	34966.24	93687.57	37322.17	267937.25	106737.74	285990.35	32
29	34993.49	93677.40	37355.34	267699.51	106749.34	28767.44	31
30	35010.74	93667.22	37388.47	267462.15	106760.94	285546.09	30
31	35047.99	93657.03	37421.63	267225.16	106772.55	285325.12	29
32	35075.23	93646.83	37454.79	266988.53	106784.18	285101.92	28
33	35102.47	93636.62	37487.97	266752.27	106795.81	284880.28	27
34	35130.70	93626.40	37521.15	266516.38	106807.47	284659.41	26
35	35156.93	93616.18	37554.34	266280.85	106819.14	284438.91	25
36	35184.16	93605.95	37587.53	266045.69	106830.82	284218.77	24
37	35211.39	93595.71	37620.73	265810.89	106842.50	283988.99	23
38	35238.62	93585.46	37653.93	265576.65	106854.20	283779.58	22
39	35265.84	93575.21	37687.16	265342.38	106865.91	283560.54	21
40	35293.08	93564.95	37720.38	265108.67	106877.63	283341.85	20
41	35320.27	93554.68	37753.61	264875.31	106889.36	283123.53	19
42	35347.48	93544.40	37786.85	264642.38	106901.10	282905.56	18
43	35374.69	93534.11	37820.10	264409.69	106913.26	282687.96	17
44	35401.90	93523.82	37853.35	264177.41	106924.63	282470.71	16
45	35429.10	93513.52	37886.61	263945.49	106936.41	282253.82	15
46	35456.30	93503.21	37919.88	263713.98	106948.20	282037.29	14
47	35483.50	93492.89	37951.16	263482.71	106960.00	281821.11	13
48	35510.70	93482.56	37986.46	263251.86	106971.82	281605.29	12
49	35537.89	93472.23	38029.73	263021.36	106983.64	281389.82	11
50	35565.08	93461.89	38053.08	262791.21	106995.48	281174.71	10
51	35592.26	93451.54	38086.33	262561.41	107007.33	280959.93	9
52	35619.44	93441.18	38119.64	262331.96	107019.19	280745.14	8
53	35646.62	93430.82	38152.96	262102.86	107031.06	280531.48	7
54	35673.80	93410.45	38186.29	261874.11	107043.95	280327.77	6
55	35700.97	93401.07	38219.66	261645.71	107054.84	280104.41	5
56	35728.14	93399.68	38252.96	261417.66	107066.75	279891.40	4
57	35755.31	93389.28	38286.31	261189.95	107078.67	279678.73	3
58	35783.48	93378.87	38319.67	260952.59	107090.60	279466.41	2
59	35810.64	93368.46	38353.03	260735.58	107102.54	279154.44	1
60	35836.79	93358.04	38386.40	260508.91	107114.50	279021.81	0

Canon Sinuum, Tangentium & Secantium.

21	Sinus	Tangens.	Secans	
0	35836.79	93358.04	38386.40	260508.91
1	35863.95	93347.61	38419.78	260282.58
2	35891.10	93337.17	38453.17	260056.59
3	35918.25	93326.73	38486.56	259830.95
4	35945.40	93316.28	38519.96	259605.64
5	35972.54	93305.82	38553.37	259380.68
6	35999.68	93295.35	38586.79	259156.06
7	36026.82	93284.87	38620.21	258931.77
8	36053.95	93274.49	38653.64	258707.82
9	36081.08	93263.90	38687.08	258484.21
10	36108.21	93253.40	38720.53	258160.94
11	36135.33	93241.89	38753.98	258038.00
12	36162.46	93230.38	38787.44	257815.39
13	36189.58	93221.86	38820.91	257593.12
14	36216.69	93211.33	38854.39	257371.18
15	36243.80	93200.79	38887.87	257149.57
16	36270.91	93190.24	38921.36	256928.30
17	36298.02	93179.68	38954.86	256707.35
18	36325.12	93169.12	38988.37	256486.74
19	36352.23	93158.55	39021.89	256268.49
20	36379.32	93147.97	39055.41	256046.49
21	36406.41	93137.38	39088.94	255826.86
22	36433.50	93126.79	39121.48	255607.56
23	36460.59	93116.19	39156.02	255388.58
24	36487.68	93105.58	39189.57	255169.92
25	36514.76	93094.96	39223.13	254951.60
26	36541.84	93084.33	39256.70	254733.59
27	36568.92	93073.70	39290.28	254518.91
28	36595.99	93063.06	39323.86	254298.55
29	36623.06	93052.41	39357.45	254081.51
30	36650.13	93041.75	39391.05	253864.79
31	36677.19	93031.09	39424.66	253648.39
32	36704.25	93020.42	39458.27	253432.31
33	36731.31	93009.74	39491.89	253210.55
34	36758.36	93099.05	39525.52	253001.11
35	36785.41	93088.35	39559.16	252783.98
36	36812.46	92977.65	39592.80	252571.17
37	36839.50	92966.94	39626.45	252356.67
38	36866.54	92956.22	39660.11	252142.49
39	36893.58	92945.49	39693.78	251928.63
40	36920.62	92934.75	39727.46	251715.07
41	36947.61	92934.01	39761.14	251501.83
42	36974.68	92913.26	39794.83	251288.90
43	37001.70	92902.50	39828.53	251076.29
44	37028.72	92891.73	39862.24	250863.98
45	37055.74	92880.95	39895.96	250651.98
46	37082.76	92870.17	39929.68	250440.29
47	37109.77	92859.38	39963.41	250228.91
48	37136.78	92848.58	39997.15	250017.84
49	37163.79	92837.77	40030.89	249807.07
50	37190.80	92826.96	40064.65	249596.61
51	37217.80	92816.14	40098.41	249386.45
52	37244.80	92805.31	40132.18	249176.60
53	37271.79	92794.47	40165.96	248967.06
54	37298.78	92783.62	40199.75	248757.81
55	37325.77	92772.77	40233.54	248548.87
56	37352.75	92761.91	40267.34	248340.23
57	37379.73	92751.04	40301.19	248131.90
58	37406.71	92740.16	40334.97	247923.86
59	37433.69	92729.28	40368.79	247716.12
60	37460.66	92718.39	40402.62	247508.69

Canon Sinuum, Tangentium & Secantium.

22	Sinus	Tangens	Secans				
0	37460.66	92918.99	40402.62	247508.89	107853.47	266946.74	60
1	37487.63	92707.49	40436.46	247301.55	107866.16	266754.67	59
2	37514.59	92696.58	40470.33	247094.70	107878.85	266562.92	58
3	37541.56	92685.66	40504.17	246888.16	107891.56	266371.43	57
4	37568.52	92674.73	40538.04	246681.91	107904.87	266180.33	56
5	37595.47	92663.80	40571.91	246475.96	107917.00	265989.47	55
6	37622.43	92652.86	40605.79	246270.30	107929.75	265798.91	54
7	37649.38	92641.91	40639.68	246064.94	107941.50	265608.63	53
8	37676.32	92630.96	40673.58	245859.87	107955.27	265418.68	52
9	37703.27	92620.00	40707.48	245655.09	107968.05	265229.02	51
10	37730.21	92609.03	40741.39	245450.61	107980.84	265039.62	50
11	37757.14	92598.09	40775.31	245246.42	107993.64	264850.54	49
12	37784.08	92587.06	40809.24	245042.52	108006.46	264661.74	48
13	37811.01	92576.06	40843.18	244838.91	108019.28	264473.23	47
14	37837.94	92565.06	40877.13	244635.59	108032.13	264285.02	46
15	37864.86	92554.05	40911.08	244432.56	108044.97	264097.09	45
16	37891.78	92543.05	40945.04	244239.82	108057.84	263909.46	44
17	37918.70	92532.00	40979.01	244037.36	108070.71	263722.11	43
18	37945.63	92520.97	41013.99	243825.19	108083.60	263535.05	42
19	37972.53	92509.94	41046.99	243623.31	108096.50	263348.28	41
20	37999.44	92498.88	41080.97	243421.72	108109.42	263161.80	40
21	38026.34	92487.82	41114.97	243220.41	108122.34	262975.60	39
22	38053.24	92476.75	41148.98	243019.38	108135.28	262780.69	38
23	38080.14	92465.68	41183.00	242818.64	108148.23	262604.06	37
24	38107.04	92454.60	41217.03	242618.19	108161.19	262418.71	36
25	38133.93	92443.51	41251.06	242418.01	108174.17	262243.66	35
26	38160.82	92432.41	41285.10	242218.12	108187.15	262048.88	34
27	38187.70	92421.31	41319.15	242018.51	108200.15	261864.39	33
28	38214.59	92410.20	41353.21	241819.18	108213.16	261680.18	32
29	38241.47	92399.08	41387.28	241620.13	108226.18	261496.24	31
30	38268.34	92387.95	41421.36	241421.36	108239.22	261312.59	30
31	38295.22	92376.81	41455.44	241222.86	108252.27	261129.22	29
32	38322.09	92365.62	41489.53	241024.65	108265.33	260946.13	28
33	38348.95	92354.52	41523.63	240826.72	108278.40	260793.32	27
34	38375.82	92343.36	41557.74	240629.06	108291.49	260580.78	26
35	38402.68	92332.19	41591.86	240433.68	108304.58	260398.52	25
36	38429.53	92321.02	41625.99	240234.57	108317.69	260216.54	24
37	38456.39	92309.84	41660.13	240037.74	108330.81	260034.84	23
38	38483.24	92298.65	41694.26	239841.18	108343.95	259853.41	22
39	38510.08	92287.45	41728.41	239644.00	108357.09	259672.31	21
40	38536.93	92276.24	41762.57	239448.89	108370.25	259491.37	20
41	38563.77	92265.03	41796.74	239253.15	108383.43	259310.97	19
42	38590.60	92253.81	41830.91	239057.69	108396.61	259130.43	18
43	38617.44	92242.58	41865.06	238862.50	108409.80	258930.37	17
44	38644.27	92231.34	41899.28	238667.58	108423.01	258770.58	16
45	38671.10	92220.09	41933.48	238472.99	108436.23	258591.07	15
46	38697.92	92208.84	41967.69	238285.55	108449.47	258411.82	14
47	38724.74	92197.58	42001.91	238084.44	108462.71	258232.84	13
48	38751.56	92186.32	42036.13	237890.60	108475.97	258054.14	12
49	38778.37	92175.03	42070.36	237697.03	108489.24	257875.70	11
50	38805.18	92163.75	42104.60	237503.72	108502.52	257697.53	10
51	38831.99	92151.46	42138.85	237310.68	108515.82	257519.63	9
52	38858.80	92141.16	42173.11	237117.91	108529.13	257345.99	8
53	38885.60	92129.85	42207.38	236925.40	108542.45	257164.62	7
54	38912.39	92118.54	42241.66	236733.16	108555.78	256987.52	6
55	38939.19	92107.22	42275.94	236541.18	108569.12	256810.69	5
56	38965.98	92095.89	42310.23	236349.46	108582.48	256634.12	4
57	38991.77	92084.55	42344.53	236158.01	108595.85	256497.81	3
58	39019.55	92073.20	42378.84	235966.83	108609.24	256284.76	2
59	39046.33	92061.85	42413.16	235777.90	108622.63	256105.99	1
60	39073.11	92050.49	42447.49	235585.24	108636.04	255930.47	0

Canon Sinuum, Tangentium & Secantium.

Z	Sinus	Tangens	Secans	
1	90673.11	92050.49	42447.49	295930.47
2	92099.89	92039.12	42481.82	255755.21
3	92126.66	92020.74	42516.16	255580.22
4	92153.43	92016.35	42550.51	—
5	92180.19	92004.96	42584.87	108676.34
6	92206.95	91993.56	42619.24	255405.48
7	92233.71	91982.85	42653.62	108716.75
8	92260.47	91970.73	42688.00	254709.15
9	92287.23	91959.31	42722.39	108743.75
10	92313.97	91947.88	42756.79	108777.27
11	92340.71	91936.44	42791.20	254189.61
12	92367.45	91924.99	42815.68	108784.35
13	92394.19	91913.53	42850.05	254362.53
14	92420.93	91902.07	42884.49	108811.48
15	92447.66	91890.60	42918.94	253825.06
16	92474.39	91879.13	42953.39	108838.66
17	92501.11	91867.63	42989.85	253328.83
18	92527.83	91856.14	43024.32	108852.27
19	92554.55	91844.64	43066.80	253157.44
20	92581.27	91833.18	43101.29	108865.89
21	92607.98	91821.61	43135.79	108879.52
22	92634.69	91810.08	43170.20	252644.78
23	92661.39	91798.55	43204.61	108893.17
24	92688.09	91787.01	43239.03	252304.26
25	92714.79	91775.46	43273.46	108902.50
26	92741.48	91763.90	43308.40	252134.38
27	92768.17	91752.34	43342.95	108917.88
28	92794.86	91740.77	43377.51	251904.75
29	92821.55	91729.19	43412.08	108931.31
30	92848.23	91717.60	43446.66	108939.04
31	92874.91	91706.01	43481.24	108956.79
32	92901.58	91694.41	43515.83	108973.41
33	92928.25	91682.80	43550.43	108974.72
34	92954.93	91671.18	43585.04	108991.11
35	92981.58	91659.53	43610.66	109008.38
36	40008.24	91647.91	43644.29	109025.03
37	40034.90	91636.37	43688.93	109042.79
38	40061.56	91624.63	43723.58	109050.57
39	40088.21	91612.96	43758.23	109058.36
40	40114.86	91601.30	43792.89	109068.76
41	40141.50	91589.63	43827.56	109082.67
42	40168.14	91577.95	43862.24	109106.53
43	40194.78	91566.26	43896.93	109126.33
44	40221.41	91554.56	43931.63	109146.48
45	40248.04	91543.86	43966.34	109166.45
46	40274.67	91531.15	44001.06	109182.43
47	40301.39	91519.43	44035.78	109200.07
48	40327.91	91507.70	44070.51	109209.09
49	40354.53	91495.96	44105.25	109226.35
50	40381.14	91484.22	44140.00	109255.84
51	40407.75	91473.47	44174.76	109273.57
52	40434.36	91460.71	44209.53	109305.53
53	40460.96	91448.97	44244.31	109317.73
54	40487.56	91437.18	44279.10	109340.10
55	40514.16	91425.40	44313.90	109366.23
56	40540.75	91413.61	44348.71	109385.72
57	40567.34	91401.81	44383.53	109408.85
58	40593.93	91389.00	44418.35	109432.21
59	40620.51	91378.19	44453.18	109455.80
60	40647.09	91366.37	44488.03	109481.46
	40673.66	91354.54	44522.87	109503.68

Canon Sinuum, Tangentium & Secantium.

24	Sinus	Tangens	Secans	
0	40673.66	91354.54	44522.87	224603.68
1	40700.23	91341.71	44557.73	224427.96
2	40726.80	91330.87	44592.60	224251.47
3	40753.37	91319.02	44619.48	224077.21
4	40779.93	91307.16	44662.37	223901.18
5	40806.49	91295.19	44697.27	223727.38
6	40833.05	91283.42	44732.17	223552.80
7	40859.60	91271.54	44767.08	223378.45
8	40886.15	91259.65	44802.00	223204.33
9	40912.69	91247.75	44836.93	223030.43
10	40939.23	91235.84	44871.87	222856.76
11	40965.77	91223.93	44906.82	222683.31
12	40992.30	91212.01	44941.78	222510.09
13	41018.83	91200.08	44976.75	222337.09
14	41045.36	91188.14	45011.73	222164.32
15	41071.89	91176.20	45046.72	221991.77
16	41098.41	91164.25	45081.72	221819.44
17	41124.93	91152.29	45116.72	221647.33
18	41151.44	91140.32	45151.74	221475.45
19	41177.95	91128.35	45186.76	221303.79
20	41204.46	91116.37	45221.79	221132.34
21	41230.96	91104.38	45256.83	220961.12
22	41257.46	91092.38	45291.88	220790.12
23	41283.95	91080.38	45326.94	220619.34
24	41310.44	91068.37	45362.01	220448.78
25	41336.93	91056.35	45397.09	220278.43
26	41363.42	91044.32	45432.18	220108.31
27	41389.90	91032.28	45467.28	219938.40
28	41416.38	91020.24	45502.39	219768.71
29	41442.85	91008.19	45537.51	219599.23
30	41469.32	90996.13	45572.64	219429.97
31	41495.79	90984.06	45607.77	219260.93
32	41522.26	90971.98	45642.91	219092.10
33	41548.78	90959.90	45678.06	218923.49
34	41575.18	90947.81	45713.22	218755.10
35	41601.63	90935.71	45748.39	218586.91
36	41628.08	90923.62	45783.57	218418.94
37	41654.53	90911.50	45818.76	218251.19
38	41680.97	90899.38	45853.96	218083.64
39	41707.41	90887.25	45889.17	217916.31
40	41733.85	90875.11	45924.39	217749.20
41	41760.28	90862.97	45959.62	217582.29
42	41786.71	90850.82	45994.86	217415.59
43	41813.13	90838.66	46030.11	217249.11
44	41839.55	90826.49	46065.37	217082.83
45	41865.97	90814.32	46100.64	216916.77
46	41892.39	90802.14	46135.91	216750.91
47	41918.80	90789.95	46171.19	216585.27
48	41945.21	90777.75	46206.48	216419.83
49	41971.61	90765.54	46241.78	216254.60
50	41998.01	90753.33	46277.09	216089.58
51	42024.41	90741.11	46312.42	215984.76
52	42050.80	90728.88	46347.96	215760.15
53	42077.19	90715.64	46383.11	215595.75
54	42103.58	90704.40	46418.46	215431.56
55	42129.96	90692.15	46453.82	215267.57
56	42156.34	90679.89	46489.19	215103.78
57	42181.73	90667.62	46524.57	214940.20
58	42209.09	90655.35	46559.96	214776.83
59	42235.46	90643.07	46595.36	214613.66
60	42261.83	90630.78	46630.77	214450.69

Canon Sinuum, Tangentium & Secantium.

25	Sinus	Tangens	Secans	
0	43161.83	90630.78	46630.77	214450.69
1	42288.19	90618.48	46666.19	214187.93
2	43314.55	90606.17	46701.61	214125.37
3	42340.90	90593.86	46737.06	213963.01
4	43367.25	90581.54	46772.51	213800.85
5	43393.60	90569.21	46807.97	213638.89
6	43419.94	90556.88	46843.43	213477.14
7	42446.28	90544.54	46878.90	213315.59
8	43472.62	90532.19	46914.38	213154.33
9	43498.95	90519.83	46949.88	212993.08
10	43525.28	90507.46	46985.39	212832.13
11	43551.61	90495.09	47020.90	212671.37
12	43577.93	90482.71	47056.43	212510.82
13	42604.25	90470.31	47091.96	212350.46
14	42630.56	90457.93	47127.51	212190.30
15	42656.87	90445.51	47163.06	212030.34
16	42683.18	90433.10	47198.63	211870.57
17	42709.49	90420.68	47234.20	211714.01
18	42735.79	90408.25	47269.78	211551.64
19	42762.09	90395.82	47305.38	211392.46
20	42788.38	90381.38	47340.98	211233.48
21	42814.67	90370.93	47376.59	211074.70
22	42840.95	90358.47	47412.23	210916.11
23	42867.23	90346.00	47447.85	210757.71
24	42893.51	90333.53	47483.49	210599.51
25	42919.79	90321.05	47519.14	210441.50
26	42946.06	90308.56	47554.81	210183.69
27	42972.33	90296.06	47590.48	210126.07
28	42998.59	90283.56	47626.16	209968.64
29	43024.85	90271.05	47661.85	209811.40
30	43051.11	90258.53	47697.55	209654.36
31	43077.36	90246.00	47733.26	209497.51
32	43103.61	90233.47	47768.99	209340.84
33	43129.86	90210.93	47804.73	209184.37
34	43156.10	90208.38	47840.46	209028.09
35	43182.34	90195.81	47876.21	208871.00
36	43208.57	90183.25	47911.97	208716.10
37	43234.80	90170.68	47947.74	208560.39
38	43261.03	90158.10	47983.52	208404.86
39	43287.26	90145.51	48019.33	208249.53
40	43313.48	90132.91	48055.13	208094.38
41	43339.70	90120.31	48090.93	207939.42
42	43365.91	90107.70	48136.75	207784.65
43	43392.12	90095.08	48162.58	207630.07
44	43418.33	90082.45	48198.42	207475.67
45	43444.13	90069.81	48234.27	207321.46
46	43470.73	90057.18	48270.14	207167.43
47	43496.92	90044.53	48306.01	207013.59
48	43523.11	90031.87	48341.89	206859.93
49	43549.30	90019.21	48377.28	206706.46
50	43575.48	90006.54	48413.68	206553.18
51	43601.66	89993.86	48449.59	206400.08
52	43627.84	89981.17	48485.52	206347.16
53	43654.01	89968.48	48521.45	206094.42
54	43680.18	89955.78	48557.39	205941.87
55	43706.34	89943.07	48593.34	205789.50
56	43732.50	89930.35	48629.31	205637.32
57	43758.66	89917.63	48665.23	205485.31
58	43784.82	89904.89	48701.26	205323.49
59	43810.97	89892.15	48737.26	205181.84
60	43837.11	89879.40	48773.26	205030.38

Canon Sinuum, Tangentium & Secantium.

26	Sinus	Tangens	Secans	
0	43837.11	89879.40	48773.26	205030.38
1	43863.26	89866.65	48809.27	204879.10
2	43880.40	89853.89	48845.35	204718.00
3	43915.53	89841.12	48884.33	204577.08
4	43941.66	89828.34	48917.37	204426.34
5	43967.79	89815.55	48953.43	204275.78
6	43993.92	89802.76	48989.49	204125.40
7	44020.04	89789.96	49025.57	203975.19
8	44046.16	89777.15	49061.65	203825.17
9	44072.27	89764.33	49097.75	203675.32
10	44098.38	89751.51	49133.86	203525.65
11	44124.48	89738.68	49169.97	203376.15
12	44150.58	89725.84	49206.10	203226.83
13	44176.68	89712.99	49242.24	203077.69
14	44202.78	89700.13	49278.38	202928.73
15	44228.87	89687.27	49314.54	202779.94
16	44254.96	89674.40	49350.71	202621.33
17	44281.04	89661.52	49386.89	202481.89
18	44307.12	89648.64	49423.08	202334.62
19	44333.20	89635.75	49459.25	202186.53
20	44359.27	89622.85	49495.49	202038.62
21	44385.34	89609.94	49531.71	201890.88
22	44411.40	89597.03	49567.94	201743.31
23	44437.46	89584.11	49604.18	201595.92
24	44463.53	89571.18	49640.43	201448.69
25	44489.57	89558.24	49676.69	201301.64
26	44515.62	89545.29	49712.97	201154.77
27	44541.67	89532.36	49749.25	201008.06
28	44567.71	89519.40	49785.74	200861.53
29	44593.75	89506.41	49821.85	200715.16
30	44619.78	89493.45	49858.16	200568.97
31	44645.81	89480.45	49894.49	200422.95
32	44671.84	89467.46	49930.82	200277.10
33	44697.86	89454.46	49967.17	200131.41
34	44723.88	89441.45	50003.52	199985.95
35	44749.90	89428.44	50039.89	199840.56
36	44775.91	89415.42	50076.27	199695.39
37	44801.92	89402.39	50112.66	199550.38
38	44827.92	89389.36	50149.06	199405.54
39	44853.93	89376.32	50187.47	199260.87
40	44879.93	89363.27	50221.89	199116.37
41	44905.91	89350.21	50258.32	198972.04
42	44931.90	89337.14	50294.76	198817.87
43	44957.89	89324.06	50331.21	198688.87
44	44983.87	89310.98	50367.67	198540.03
45	45009.85	89297.89	50404.15	198396.36
46	45035.83	89284.79	50440.63	198253.86
47	45061.79	89271.69	50477.13	198109.52
48	45087.76	89258.58	50513.63	197966.35
49	45113.72	89245.46	50550.15	197823.34
50	45139.68	89232.34	50586.68	197680.50
51	45165.63	89219.20	50623.23	197537.82
52	45191.58	89206.06	50659.77	197395.31
53	45217.53	89193.91	50696.33	197253.96
54	45243.47	89179.77	50732.90	197110.77
55	45269.41	89166.59	50769.48	196968.74
56	45295.35	89153.45	50806.07	196816.88
57	45321.28	89140.24	50842.67	196681.18
58	45347.21	89127.05	50879.28	196543.64
59	45373.13	89113.85	50915.91	196402.27
60	45399.05	89100.63	50952.54	196161.05

Canon Sinuum, Tangentium & Secantium.

27	Sinns	Tangens	Secans	
0	46399.05	89000.67	50978.54	196261.05
1	46424.97	89037.44	50989.19	196120.00
2	46450.88	89074.22	51025.85	195979.10
3	46476.79	89068.00	51062.52	195838.37
4	46502.69	89047.77	51099.19	195697.80
5	46528.59	89034.53	51135.88	195557.39
6	46554.49	89021.28	51172.59	195417.13
7	46580.38	89008.02	51209.30	195277.04
8	46606.27	88994.76	51246.02	195137.11
9	46632.16	88981.49	51282.75	194997.33
10	46658.04	88968.21	51319.50	194857.71
11	46683.92	88954.93	51356.25	194718.16
12	46709.79	88941.64	51393.02	194578.96
13	46735.66	88928.34	51429.80	194439.81
14	46761.53	88915.03	51466.58	194300.83
15	46787.39	88901.73	51503.38	194162.00
16	46813.25	88888.39	51540.19	194023.33
17	46839.10	88875.06	51577.02	193884.81
18	46864.95	88861.72	51613.85	193746.45
19	46890.80	88848.37	51650.69	193608.25
20	46916.64	88835.02	51687.55	193470.20
21	46942.43	88821.66	51724.41	193332.31
22	46968.32	88808.49	51761.30	193194.57
23	47094.15	88794.92	51798.18	193056.98
24	47099.98	88781.54	51835.08	192919.16
25	47045.80	88768.15	51871.99	192782.28
26	47071.62	88754.75	51908.91	192645.16
27	47097.44	88741.34	51945.84	192508.19
28	47123.25	88727.93	51982.78	192371.38
29	47149.06	88714.51	52019.74	192234.72
30	47174.86	88701.05	52056.70	192098.21
31	47200.66	88687.64	52093.68	191961.86
32	47226.46	88674.20	52130.67	191825.65
33	47252.25	88660.75	52167.67	191689.65
34	47278.04	88647.39	52204.63	191533.70
35	47303.83	88633.83	52241.70	191417.95
36	47329.60	88620.36	52278.74	191282.36
37	47355.38	88606.88	52315.78	191146.91
38	47381.15	88593.39	52352.84	191011.62
39	47406.93	88579.89	52389.90	190876.47
40	47432.69	88566.49	52426.98	190741.47
41	47458.47	88552.88	52464.07	190606.63
42	47484.21	88539.76	52501.17	190471.93
43	47509.96	88525.63	52538.29	190337.38
44	47535.71	88512.30	52575.41	190202.99
45	47561.45	88498.76	52612.54	190068.74
46	47587.19	88485.21	52649.69	189934.64
47	47612.93	88471.66	52686.85	189804.68
48	47638.66	88458.10	52724.02	189666.88
49	47664.39	88444.53	52761.20	189533.22
50	47690.12	88430.95	52798.39	189399.71
51	47715.84	88417.56	52835.59	189266.34
52	47741.56	88403.77	52872.81	189133.13
53	47767.27	88390.17	52910.04	189000.06
54	47792.98	88376.56	52947.27	188867.13
55	47818.69	88362.94	52984.52	188734.36
56	47844.39	88349.31	53021.78	188601.72
57	47870.09	88335.69	53059.06	188469.24
58	47895.78	88322.05	53096.34	188336.90
59	47921.47	88308.41	53133.64	188204.22
60	47947.16	88294.76	53170.94	188072.65

Canon Sinuum, Tangentium & Secantium.

28	Sinus	Tangens	Secans	
0	46947. 16	88294. 76	53170. 94	188072. 65
1	46972. 84	88181. 10	53208. 36	187940. 74
2	46998. 52	88267. 43	53245. 59	187808. 98
3	47014. 19	88153. 75	53282. 93	187677. 36
4	47049. 86	88240. 07	53320. 29	187545. 38
5	47075. 53	88226. 38	53357. 65	187414. 55
6	47101. 19	88121. 68	53395. 03	187283. 36
7	47126. 85	88198. 98	53432. 41	187152. 31
8	47152. 50	88181. 27	53469. 83	187021. 41
9	47178. 15	88171. 55	53507. 23	186890. 64
10	47203. 80	88176. 82	53544. 05	186760. 03
11	47229. 44	88144. 09	53581. 08	186629. 55
12	47255. 08	88130. 35	53619. 53	186499. 21
13	47280. 71	88116. 60	53656. 99	186369. 02
14	47306. 34	88102. 84	53694. 46	186238. 96
15	47331. 97	88089. 07	53731. 94	186109. 05
16	47357. 59	88075. 30	53769. 43	185979. 28
17	47383. 21	88061. 52	53806. 94	185849. 65
18	47408. 83	88047. 73	53844. 45	185710. 19
19	47434. 43	88033. 94	53881. 98	185580. 80
20	47460. 04	88020. 14	53919. 52	185461. 59
21	47485. 64	88006. 33	53957. 07	185332. 52
22	47511. 24	87992. 51	53994. 64	185103. 38
23	47536. 83	87978. 69	54021. 21	185074. 79
24	47562. 41	87964. 86	54059. 80	184946. 13
25	47588. 01	87951. 02	54107. 40	184817. 61
26	47613. 59	87937. 17	54145. 01	184689. 23
27	47639. 17	87923. 32	54182. 63	184560. 99
28	47664. 74	87909. 46	54220. 17	184432. 89
29	47690. 31	87895. 59	54257. 91	184304. 92
30	47715. 88	87881. 71	54295. 57	184177. 09
31	47741. 44	87867. 83	54333. 24	184049. 39
32	47767. 00	87853. 94	54370. 92	183911. 84
33	47792. 55	87840. 04	54408. 62	183794. 42
34	47818. 10	87826. 13	54446. 32	183667. 13
35	47843. 64	87812. 22	54484. 04	183539. 99
36	47869. 18	87798. 30	54511. 77	183412. 97
37	47894. 73	87784. 37	54559. 51	183286. 10
38	47920. 26	87770. 43	54597. 26	183159. 36
39	47945. 79	87756. 49	54635. 03	183032. 75
40	47971. 31	87742. 54	54671. 81	182906. 28
41	47996. 83	87718. 58	54710. 60	182779. 94
42	48022. 35	87714. 61	54748. 40	182653. 74
43	48047. 86	87700. 64	54786. 21	182527. 67
44	48073. 37	87686. 66	54824. 04	182401. 73
45	48098. 88	87672. 67	54861. 88	182275. 93
46	48124. 38	87658. 68	54899. 73	182150. 26
47	48149. 88	87644. 68	54937. 59	182024. 73
48	48175. 37	87630. 67	54975. 46	181899. 32
49	48200. 86	87616. 65	55013. 35	181774. 05
50	48226. 34	87602. 62	55051. 25	181648. 92
51	48251. 83	87588. 59	55089. 16	181523. 91
52	48277. 30	87574. 55	55127. 08	181399. 04
53	48302. 77	87560. 50	55165. 02	181274. 10
54	48328. 24	87546. 45	55202. 97	181149. 69
55	48353. 70	87532. 39	55240. 93	181025. 21
56	48379. 16	87518. 32	55278. 90	180900. 86
57	48404. 62	87504. 24	55316. 88	180776. 64
58	48430. 07	87490. 16	55354. 88	180652. 56
59	48455. 51	87476. 07	55392. 88	180538. 60
60	48480. 96	87461. 97	55430. 90	180424. 78

Canon Sinuum, Tangentium & Secantium.

29	Sinus	Tangens	Secans	
0	48480.96	87461.97	55430.90	180404.78
1	48506.40	87447.86	55468.94	180181.08
2	48531.84	87433.75	55506.98	180157.51
3	48557.27	87419.63	55545.04	180034.08
4	48583.70	87405.50	55583.11	179910.77
5	48608.12	87391.36	55621.19	179787.79
6	48633.54	87377.22	55659.29	179664.54
7	48658.95	87361.07	55697.39	179541.62
8	48684.36	87348.91	55735.51	179418.83
9	48709.77	87334.75	55773.64	179296.16
10	48735.17	87320.58	55811.79	179173.62
11	48760.57	87306.40	55849.94	179051.21
12	48785.97	87292.21	55888.11	178928.98
13	48811.36	87278.01	55926.29	178806.78
14	48836.74	87263.81	55964.48	178684.75
15	48862.12	87249.60	56000.69	178562.85
16	48887.50	87235.38	56040.91	178441.07
17	48912.87	87221.16	56079.14	178319.43
18	48938.24	87206.93	56117.38	178197.90
19	48963.61	87191.69	56155.64	178076.51
20	48988.97	87178.44	56193.91	177955.24
21	49014.33	87164.19	56232.19	177834.09
22	49039.68	87149.93	56270.48	177713.07
23	49065.03	87135.66	56308.79	177592.18
24	49090.37	87121.38	56347.10	177471.41
25	49115.71	87107.10	56385.43	177350.76
26	49141.05	87093.81	56423.78	177230.24
27	49166.38	87078.51	56462.13	177109.81
28	49191.71	87064.20	56500.50	176989.58
29	49217.04	87049.89	56538.88	176869.43
30	49243.36	87035.57	56577.28	176749.40
31	49267.67	87021.24	56615.68	176629.50
32	49292.98	87006.90	56654.10	176509.73
33	49318.29	86992.56	56692.53	176390.07
34	49343.59	86978.21	56730.98	176270.53
35	49368.89	86963.85	56769.44	176151.12
36	49394.19	86949.49	56807.91	176031.83
37	49419.48	86935.12	56846.39	175913.67
38	49444.77	86920.74	56884.88	175793.62
39	49470.05	86906.35	56923.39	175674.70
40	49495.33	86891.96	56961.91	175555.90
41	49520.60	86877.56	57000.45	175437.22
42	49545.87	86863.15	57038.99	175318.66
43	49571.13	86848.73	57077.55	175200.43
44	49596.39	86834.31	57116.12	175081.91
45	49621.65	86819.88	57154.71	174963.71
46	49646.90	86805.44	57193.31	174845.64
47	49672.15	86791.00	57231.92	174727.68
48	49697.40	86776.55	57270.54	174609.84
49	49722.64	86761.09	57309.18	174492.13
50	49747.87	86747.62	57347.83	174374.53
51	49773.10	86733.14	57386.49	174257.05
52	49798.33	86718.66	57425.16	174139.69
53	49823.55	86704.17	57463.85	174022.45
54	49848.77	86689.67	57502.55	173905.33
55	49873.99	86675.17	57541.26	173788.33
56	49899.20	86660.66	57579.99	173671.44
57	49924.41	86646.14	57618.73	173554.68
58	49949.61	86631.61	57657.48	173438.03
59	49974.81	86617.08	57696.25	173321.49
60	50000.00	86602.54	57735.03	173205.08

Canon Sinuum, Tangentium & Secantium.

30	Sinus	Tangens	Secans			
0	50000.00	86602.54	57735.08	115470.05	200000.00	60
1	50025.19	86587.99	57723.82	115489.45	199899.29	59
2	50050.38	86573.43	57812.62	115508.87	199798.70	58
3	50075.56	86558.87	57811.44	115528.30	199698.23	57
4	50100.74	86544.30	57809.27	115547.75	199597.88	56
5	50125.91	86525.72	57929.11	115567.22	199497.64	55
6	50151.08	86515.14	57906.97	115586.70	199397.13	54
7	50176.24	86500.55	58006.84	115606.20	199297.53	53
8	50201.40	86485.95	58045.73	115625.72	199197.64	52
9	50226.55	86471.34	58084.62	115645.23	199097.87	51
10	50251.70	86456.73	58123.53	115664.80	198998.22	50
11	50276.85	86442.11	58162.45	115684.36	198988.69	49
12	50301.99	86427.48	58201.39	115703.94	198799.27	48
13	50327.13	86412.84	58240.30	115723.54	198699.97	47
14	50352.27	86398.20	58279.30	115743.15	198600.60	46
15	50377.40	86383.55	58318.38	115762.78	198501.72	45
16	50402.53	86368.89	58357.37	115782.43	198402.76	44
17	50427.65	86354.23	58396.27	115802.09	198303.93	43
18	50452.77	86339.56	58435.28	115821.77	198205.20	42
19	50477.88	86324.88	58474.31	115841.47	198106.59	41
20	50502.99	86310.19	58513.35	115861.18	198008.10	40
21	50528.09	86295.49	58552.41	115880.91	197909.72	39
22	50553.19	86280.79	58591.48	115900.67	197811.46	38
23	50578.28	86266.08	58630.56	115920.41	197713.31	37
24	50603.37	86251.36	58669.65	115940.19	197615.27	36
25	50618.46	86236.64	58708.76	115959.99	197517.35	35
26	50653.55	86221.91	58747.88	115979.80	197419.54	34
27	50678.63	86107.17	58787.02	116010.59	115999.63	33
28	50703.70	86102.43	58826.17	116019.47	197214.26	32
29	50728.77	86177.68	58865.33	116039.33	197126.83	31
30	50753.84	86162.92	58904.50	116059.21	197029.44	30
31	50778.90	86148.15	58943.69	116079.11	196931.20	29
32	50803.96	86133.37	58982.89	116099.02	196835.07	28
33	50829.01	86118.59	59012.11	116118.93	196738.05	27
34	50854.06	86103.80	59051.34	116138.89	196641.14	26
35	50879.10	86089.00	59100.58	116158.85	196544.34	25
36	50904.14	86074.20	59139.83	116178.83	196447.67	24
37	50929.18	86059.39	59179.10	116198.82	196351.80	23
38	50954.21	86044.57	59218.39	116218.83	196254.64	22
39	50979.24	86029.74	59257.68	116338.86	196158.39	21
40	51004.26	86014.91	59296.99	116458.91	196062.06	20
41	51029.28	86000.07	59336.32	116578.97	195965.93	19
42	51054.29	85985.22	59375.66	116399.05	195869.92	18
43	51079.30	85970.37	59415.01	116419.14	195774.01	17
44	51104.31	85955.51	59454.37	116519.25	195678.23	16
45	51129.31	85940.64	59493.75	116639.38	195582.54	15
46	51154.31	85925.76	59533.14	116759.53	195486.97	14
47	51179.30	85910.88	59572.54	116862.56	195399.69	13
48	51204.29	85895.99	59611.96	116984.87	195296.15	12
49	51229.27	85881.09	59651.40	117040.67	195200.91	11
50	51254.25	85866.18	59690.84	117059.88	195105.77	10
51	51279.23	85851.27	59730.30	117419.21	195010.75	9
52	51304.19	85836.35	59769.78	117308.64	194915.83	8
53	51329.16	85821.42	59809.27	117198.18	194821.02	7
54	51354.12	85806.49	59848.77	117087.82	194726.32	6
55	51379.08	85791.55	59888.28	116677.58	194631.73	5
56	51404.04	85775.60	59927.81	116667.44	194537.25	4
57	51428.99	85761.64	59967.35	116657.41	194442.88	3
58	51453.93	85746.68	60006.91	116647.48	194348.61	2
59	51478.87	85731.71	60046.48	116637.66	194254.45	1
60	51503.81	85716.73	60086.06	116627.95	194160.40	0

Canon Sinuum, Tangentium & Secantium.

31	Sinus	Tangens	Secans	
9	51503.81	85716.73	60086.06	166427.95
1	51526.74	85701.74	60125.66	166318.34
2	51553.67	85686.75	60167.27	166206.84
3	51578.59	85671.75	60204.90	166099.45
4	51603.51	85656.74	60244.54	165990.16
5	51628.43	85641.73	60284.19	165880.97
6	51653.33	85626.71	60323.86	165771.89
7	51678.24	85611.68	60363.54	165662.91
8	51703.14	85596.64	60403.23	165554.05
9	51728.04	85581.60	60442.94	165445.29
10	51753.93	85566.55	60481.66	165336.63
11	51777.82	85551.49	60521.40	165228.08
12	51802.70	85536.42	60561.15	165119.63
13	51827.58	85521.35	60601.92	165011.28
14	51852.46	85506.27	60641.70	164903.04
15	51877.33	85491.18	60681.49	164794.90
16	51902.19	85476.09	60721.30	164686.86
17	51927.05	85460.99	60761.12	164578.93
18	51951.91	85445.88	60800.95	164471.11
19	51976.76	85430.76	60840.80	164363.38
20	52001.61	85415.64	60880.67	164255.76
21	52026.46	85400.51	60920.54	164148.24
22	52051.30	85385.37	60960.43	164040.82
23	52076.13	85370.23	61000.34	163933.51
24	52100.96	85355.08	61040.26	163826.30
25	52125.79	85339.92	61080.19	163719.19
26	52150.61	85324.75	61120.14	163614.18
27	52175.43	85309.58	61160.11	163505.28
28	52200.24	85294.40	61200.08	163398.47
29	52225.05	85279.21	61240.07	163293.77
30	52249.86	85264.03	61280.08	163185.17
31	52274.66	85248.81	61320.10	163078.67
32	52299.45	85233.60	61360.13	163072.27
33	52324.24	85218.38	61400.18	162865.97
34	52349.03	85203.16	61440.24	162759.72
35	52373.81	85187.93	61480.32	162653.68
36	52398.59	85178.69	61520.41	162547.68
37	52423.36	85157.44	61560.52	162441.78
38	52448.13	85141.19	61600.64	162335.99
39	52472.90	85126.93	61640.77	162230.29
40	52497.66	85111.66	61680.92	162124.69
41	52522.41	85096.39	61721.08	162019.20
42	52547.16	85081.11	61761.26	161913.80
43	52571.91	85067.83	61801.45	161808.50
44	52596.65	85050.53	61841.66	161703.30
45	52621.39	85035.23	61881.88	161594.80
46	52646.12	85019.91	61921.11	161493.20
47	52670.85	85004.59	61961.36	161388.29
48	52695.58	84989.27	62002.63	161283.49
49	52720.30	84973.94	62042.91	161178.78
50	52745.02	84958.60	62083.20	161074.17
51	52769.73	84943.25	62123.51	160969.66
52	52794.44	84927.90	62163.83	160863.25
53	52819.14	84912.54	62204.17	160760.94
54	52843.84	84897.17	62244.52	160656.78
55	52868.53	84881.79	62284.88	160552.60
56	52893.22	84866.41	62325.26	160448.58
57	52917.90	84851.02	62365.66	160344.65
58	52942.58	84835.62	62406.07	160240.82
59	52967.26	84820.22	62446.50	160137.99
60	52991.93	83804.81	62486.94	160033.49

Canon Sinuum, Tangentium & Secantium.

32	Sinus	Tangens	Secans	
0	52991.98	84804.81	62486.94	160033.45
1	53016.59	84789.39	62517.39	159019.91
2	53041.25	84773.96	62567.86	159816.47
3	53065.91	84758.53	62608.34	159723.12
4	53090.56	84743.09	62648.84	159619.87
5	53115.21	84727.64	62689.35	159516.72
6	53139.86	84712.19	62729.88	159413.66
7	53164.50	84696.73	62770.43	159310.70
8	53189.13	84681.26	62810.98	159207.83
9	53213.76	84665.78	62851.56	159105.05
10	53238.39	84650.30	62894.15	159002.38
11	53263.01	84634.81	61932.75	158899.79
12	53287.63	84619.31	62973.36	158797.30
13	53312.24	84603.81	63013.99	158694.91
14	53336.85	84588.30	63054.64	158592.61
15	53361.45	84572.78	63095.30	158490.41
16	53386.05	84557.35	63135.98	158388.30
17	53410.64	84541.72	63176.67	158286.28
18	53435.23	84526.18	63217.38	158184.36
19	53459.82	84510.63	63258.10	158082.53
20	53484.40	84495.08	63298.83	157980.79
21	53508.98	84479.52	63339.58	157879.15
22	53533.55	84463.95	63380.35	157777.60
23	53558.12	84448.37	63421.13	157676.15
24	53582.68	84432.79	63461.93	157574.79
25	53607.24	84417.20	63502.74	157473.52
26	53631.79	84401.60	63543.57	157372.34
27	53656.34	84386.00	63584.41	157271.26
28	53680.88	84370.39	63625.27	157170.26
29	53705.42	84354.77	63666.14	157069.36
30	53729.96	84339.14	63707.03	156968.56
31	53754.49	84323.51	63747.93	156867.84
32	53779.02	84307.87	63788.83	156767.22
33	53803.54	84292.22	63829.78	156666.69
34	53828.06	84276.57	63870.73	156566.25
35	53852.57	84260.91	63911.69	156465.90
36	53877.08	84245.24	63952.67	156365.64
37	53901.58	84239.56	63993.66	156165.48
38	53926.08	84213.88	64034.67	156165.40
39	53950.58	84198.19	64075.69	156065.42
40	53975.07	84182.49	64116.73	155965.52
41	53999.55	84166.79	64157.79	155865.72
42	54024.03	84151.08	64198.86	155766.01
43	54048.51	84135.36	64239.95	155666.39
44	54072.98	84119.63	64281.05	155566.85
45	54097.45	84103.90	64222.16	155467.41
46	54121.91	84088.16	64363.29	155368.06
47	54146.37	84072.41	64404.44	155268.80
48	54170.84	84056.66	64445.60	155169.63
49	54195.27	84040.90	64486.78	155070.54
50	54219.71	84025.13	64527.97	154971.55
51	54244.15	84009.35	64569.18	154873.64
52	54268.59	83993.57	64610.41	154773.83
53	54293.02	83977.78	64651.65	154675.10
54	54317.44	83961.98	64693.90	154576.46
55	54341.86	83946.18	64734.17	154477.92
56	54366.28	83930.37	64775.46	154379.46
57	54390.69	83914.55	64816.76	154281.08
58	54415.10	83898.73	64858.08	154182.80
59	54439.50	83882.90	64899.41	154084.60
60	54463.90	83867.06	64940.76	153986.50

Canon Sinuum, Tangentium & Secantium;

33	Sinus	Tangens	Secans	
1	54469.90	83867.06	64940.76	133986.90
2	54486.30	83851.41	64982.12	133886.48
3	54512.69	83835.36	65023.50	133790.55
4	54536.45	83803.63	65106.81	133594.94
5	54559.83	83787.75	65147.74	133497.27
6	54610.20	83774.87	65189.18	133399.69
7	54634.56	83755.98	65230.64	133302.20
8	54658.91	83740.08	65274.11	133204.79
9	54683.28	83724.18	65313.60	133107.47
10	54707.62	83704.27	65355.11	133010.83
11	54731.98	83692.35	65396.63	132913.08
12	54756.32	83676.43	65438.17	132816.08
13	54780.66	83660.50	65479.72	132719.04
14	54804.99	83644.56	65521.29	132623.15
15	54829.32	83628.61	65563.87	132526.35
16	54853.65	83612.66	65604.47	132428.69
17	54877.97	83596.70	65646.09	132332.00
18	54902.28	83580.73	65687.72	132235.45
19	54926.59	83564.76	65729.37	132138.99
20	54950.90	83548.78	65771.03	132042.61
21	54975.20	83534.79	65812.71	131946.32
22	54999.50	83516.80	65854.41	131850.12
23	55023.79	83500.80	65896.13	131754.00
24	55048.08	83484.79	65937.85	131657.96
25	55072.36	83468.77	65979.59	131559.01
26	55096.64	83452.75	66021.35	131466.14
27	55120.91	83436.72	66063.13	131370.36
28	55145.18	83420.68	66104.92	131274.66
29	55169.44	83404.63	66146.73	131179.01
30	55193.70	83388.58	66188.56	131083.52
31	55217.95	83372.57	66230.40	130986.07
32	55242.20	83356.45	66272.26	130882.71
33	55266.47	83340.38	66314.13	130787.43
34	55290.69	83324.30	66356.02	130679.24
35	55314.92	83308.21	66397.92	130567.13
36	55339.15	83292.12	66439.84	130512.80
37	55363.38	83276.02	66481.78	130417.16
38	55387.60	83259.91	66523.73	130324.30
39	55411.82	83243.82	66565.70	130227.72
40	55436.03	83227.68	66607.69	130132.82
41	55460.24	83211.57	66649.69	130038.20
42	55484.44	83195.41	66691.71	129943.67
43	55508.64	83179.27	66733.75	129849.23
44	55532.83	83163.12	66775.80	129774.86
45	55557.01	83146.96	66817.87	129666.58
46	55581.21	83130.79	66859.95	129566.38
47	55605.39	83114.62	66901.03	129473.36
48	55629.56	83098.44	66944.17	129378.82
49	55653.73	83082.26	66986.30	129274.26
50	55677.90	83066.07	67028.45	129190.38
51	55701.06	83049.87	67070.62	129096.59
52	55726.21	83033.66	67112.80	129002.82
53	55750.36	83017.45	67155.00	128909.23
54	55774.51	83001.23	67197.21	128816.70
55	55798.65	82985.00	67230.44	128723.23
56	55822.79	82968.76	67271.69	128638.84
57	55846.92	82952.52	67323.96	128535.83
58	55871.05	82936.37	67366.24	128442.30
59	55895.17	82920.03	67408.54	128349.16
60	55919.39	82903.76	67450.85	128256.10

Canon Sinuum, Tangentium & Secantium.

34	Sinus	Tangens	Secans				
0	55919.29	82803.76	67450.85	148256.10	120621.80	178349.16	60
1	55948.40	82887.49	67493.18	148163.21	120645.48	178352.08	59
2	55967.51	82871.81	67531.53	148070.21	120669.18	178673.08	58
3	55991.61	82854.93	67577.90	147977.38	120692.89	178598.17	57
4	56015.71	82838.64	67620.28	147884.63	120716.62	178522.33	56
5	56039.81	82822.34	67663.68	147791.97	120740.37	178444.57	55
6	56063.90	82806.03	67703.09	147699.38	120764.14	178367.90	54
7	56087.98	82789.72	67747.52	147606.88	120787.98	178391.31	53
8	56111.06	82773.40	67789.97	147514.45	120811.75	178214.79	52
9	56136.14	82757.07	67833.44	147428.10	120835.59	178328.36	51
10	56160.21	82740.74	67874.93	147329.83	120859.44	178062.08	50
11	56184.28	82724.40	67917.43	147237.64	120883.31	177985.74	49
12	56208.34	82708.05	67959.93	147145.53	120907.20	177909.55	48
13	56232.39	82691.70	68002.46	147053.50	120931.12	177833.43	47
14	56256.44	82675.34	68045.01	146961.55	120955.05	177757.40	46
15	56280.49	82658.97	68087.58	146869.67	120979.00	177681.48	45
16	56304.53	82642.60	68130.16	146777.87	121001.97	177605.78	44
17	56328.57	82636.28	68172.76	146686.16	121016.96	177529.79	43
18	56352.60	82609.83	68215.38	146594.52	121050.97	177454.08	42
19	56376.63	82593.43	68258.01	146502.96	121075.00	177378.45	41
20	56400.65	82577.03	68300.66	146411.47	121099.05	177308.90	40
21	56424.67	82560.62	68343.33	146320.07	121123.11	177227.43	39
22	56448.69	82544.20	68386.01	146228.74	121147.21	177151.04	38
23	56472.70	82527.78	68428.71	146137.49	121171.32	177076.73	37
24	56496.70	82511.35	68471.43	146046.32	121195.45	177001.49	36
25	56520.70	82494.91	68514.17	145955.22	121219.60	176926.33	35
26	56544.69	82478.47	68556.93	145864.20	121243.77	176851.85	34
27	56568.68	82462.05	68599.69	145773.26	121267.96	176776.25	33
28	56592.67	82445.56	68640.47	145682.40	121295.17	176701.33	32
29	56616.65	82419.09	68685.27	145591.61	121316.40	176636.49	31
30	56640.63	82402.62	68728.10	145500.90	121340.64	176551.73	30
31	56664.59	82396.14	68770.94	145410.27	121364.91	176477.04	29
32	56688.56	82379.65	68813.79	145319.71	121389.30	176404.43	28
33	56712.53	82363.26	68856.66	145229.23	121413.51	176327.91	27
34	56736.48	82346.66	68899.55	145138.83	121437.82	176253.45	26
35	56760.43	82330.15	68942.46	145048.50	121461.18	176179.08	25
36	56784.37	82313.64	68985.38	144958.25	121486.55	176104.78	24
37	56808.31	82297.12	69028.32	144868.08	121510.94	176030.56	23
38	56832.25	82280.59	69071.28	144777.98	121535.35	175916.41	22
39	56856.18	82264.05	69114.25	144687.96	121559.78	175882.36	21
40	56880.11	82247.51	69157.24	144598.01	121584.23	175808.37	20
41	56904.03	82230.96	69200.25	144508.14	121608.70	175734.45	19
42	56927.95	82214.40	69243.28	144418.34	121631.19	175660.63	18
43	56951.86	82197.84	69286.33	144338.62	121657.70	175586.87	17
44	56975.77	82181.27	69329.39	144338.97	121688.23	175513.19	16
45	56999.68	82164.69	69372.47	144149.40	121706.78	175419.59	15
46	57013.58	82148.11	69415.17	144059.91	121731.31	175366.07	14
47	57047.47	82131.52	69458.68	143970.49	121755.94	175191.08	13
48	57071.36	82114.92	69501.81	143881.14	121780.55	175119.24	12
49	57095.24	82098.31	69544.96	143791.87	121805.28	175145.94	11
50	57119.12	82081.70	69588.13	143701.68	121829.83	175071.73	10
51	57142.99	82064.08	69631.31	143613.56	121848.50	174999.18	9
52	57166.86	82048.46	69674.51	143524.51	121879.19	174956.51	8
53	57190.73	82031.83	69717.73	143439.54	121900.90	174873.58	7
54	57214.59	82015.10	69760.99	143346.64	121928.64	174798.60	6
55	57238.44	81998.54	69804.22	143257.81	121953.39	174757.06	5
56	57262.29	81981.89	69847.49	143169.06	121978.16	174634.99	4
57	57286.14	81965.23	69880.98	143080.39	122001.96	174562.30	3
58	57309.98	81948.56	69934.69	142991.78	122025.77	174489.69	2
59	57333.81	81931.89	69977.41	142903.26	122051.60	174417.15	1
60	57357.64	81915.21	70010.75	142814.80	122077.46	174334.68	0

Canon Sinuum, Tangentium & Secantium.

35	Sinus	Tangens	Secans
0	57377.64	81915.21	70020.75
1	57381.47	81808.52	70064.21
2	57405.29	81881.82	70107.49
3	57429.11	81865.12	70150.89
4	57453.93	81848.41	70194.30
5	57476.73	81831.69	70237.73
6	57500.52	81814.97	70281.18
7	57524.32	81798.24	70324.65
8	57548.11	81781.50	70368.13
9	57571.90	81764.76	70411.63
10	57595.68	81748.01	70455.15
11	57619.46	81731.35	70498.69
12	57643.23	81714.49	70542.24
13	57667.00	81697.72	70585.81
14	57690.76	81680.94	70629.40
15	57714.52	81664.15	70673.01
16	57738.27	81647.36	70716.64
17	57763.02	81630.56	70760.29
18	57785.76	81613.76	70803.95
19	57809.50	81596.95	70847.63
20	57833.23	81580.13	70891.33
21	57856.96	81563.30	70935.05
22	57880.68	81546.47	70978.78
23	57904.40	81529.63	71022.53
24	57928.12	81512.78	71066.30
25	57951.83	81495.93	71110.09
26	57975.53	81479.06	71153.90
27	57999.23	81462.19	71197.73
28	58023.92	81445.32	71241.57
29	58046.61	81428.44	71285.43
30	58070.30	81411.55	71329.31
31	58093.98	81394.65	71373.21
32	58117.65	81377.73	71417.13
33	58141.32	81360.84	71461.06
34	58164.98	81343.93	71505.01
35	58188.64	81327.01	71548.98
36	58212.30	81310.08	71593.97
37	58235.95	81293.14	71636.98
38	58259.59	81276.20	71681.01
39	58283.23	81259.25	71725.09
40	58306.87	81242.29	71769.11
41	58330.50	81225.32	71813.19
42	58354.12	81208.35	71857.29
43	58377.74	81191.37	71901.41
44	58401.36	81174.39	71945.55
45	58424.97	81157.40	71989.70
46	58448.57	81140.40	72033.87
47	58472.17	81123.39	72078.06
48	58495.77	81106.38	72122.27
49	58519.36	81089.36	72166.50
50	58542.94	81072.33	72210.75
51	58566.52	81055.30	72255.02
52	58590.10	81038.26	72299.31
53	58613.67	81021.21	72343.61
54	58637.24	81004.16	72387.93
55	58660.80	80987.10	72424.27
56	58684.35	80970.03	72476.03
57	58707.90	80951.96	72521.01
58	58731.45	80935.88	72565.41
59	58754.99	80918.79	72609.83
60	58778.53	80901.70	72654.26

. Canon Sinuum , Tangentium & Secantium.

36	Sinus	Tangens	Secans	
0	58778. 53	80901. 70	73654. 26	137638. 19
1	58802. 06	80884. 60	72698. 71	137554. 03
2	58825. 58	80867. 49	72743. 18	137469. 94
3	58849. 10	80850. 37	72787. 67	137385. 91
4	58873. 63	80833. 25	72832. 18	137301. 95
5	58896. 13	80816. 12	72876. 71	137218. 05
6	58919. 66	80798. 99	72921. 26	137134. 23
7	58943. 14	80781. 85	72965. 82	137050. 47
8	58966. 63	80764. 70	73010. 40	136966. 78
9	58990. 12	80747. 54	73055. 01	136883. 15
10	59013. 61	80730. 38	73099. 63	136799. 19
11	59037. 09	80713. 21	73144. 27	136716. 10
12	59060. 57	80696. 03	73188. 94	136633. 67
13	59084. 04	80678. 85	73233. 62	136549. 31
14	59107. 50	80661. 66	73278. 31	136466. 03
15	59130. 96	80644. 46	73323. 03	136382. 79
16	59154. 43	80627. 26	73367. 77	136299. 63
17	59177. 87	80610. 05	73412. 53	136216. 53
18	59101. 38	80592. 83	73457. 30	136133. 50
19	59224. 76	80575. 60	73502. 10	136050. 94
20	59248. 19	80558. 37	73546. 91	135967. 64
21	59271. 62	80541. 33	73591. 74	135884. 81
22	59295. 05	80523. 89	73636. 60	135804. 04
23	59318. 47	80506. 64	73681. 47	135719. 34
24	59341. 89	80489. 38	73726. 36	135636. 70
25	59365. 30	80472. 11	73771. 27	135554. 13
26	59388. 71	80454. 84	73816. 20	135471. 62
27	59412. 11	80437. 56	73861. 15	135389. 18
28	59435. 50	80420. 28	73906. 11	135306. 80
29	59458. 89	80403. 99	73951. 10	135224. 49
30	59482. 28	80381. 69	73996. 11	135142. 24
31	59505. 66	80368. 38	74048. 16	135060. 06
32	59529. 03	80351. 07	74086. 18	134977. 94
33	59552. 40	80333. 75	74131. 24	134895. 89
34	59575. 77	80316. 42	74176. 33	134813. 90
35	59599. 13	80199. 09	74221. 43	134731. 97
36	59622. 49	80281. 75	74266. 55	134650. 11
37	59645. 84	80264. 40	74311. 70	134568. 32
38	59669. 18	80247. 05	74356. 86	134486. 58
39	59692. 51	80229. 69	74402. 04	134404. 92
40	59715. 86	80213. 31	74447. 24	134323. 31
41	59739. 19	80194. 94	74491. 46	134241. 77
42	59762. 51	80177. 56	74537. 70	134160. 29
43	59785. 83	80160. 17	74582. 96	134078. 88
44	59809. 15	80143. 78	74628. 24	133997. 53
45	59832. 46	80125. 38	74673. 54	133916. 24
46	59855. 76	80107. 97	74718. 86	133835. 02
47	59879. 06	80090. 56	74764. 20	133753. 86
48	59902. 36	80073. 14	74809. 56	133672. 76
49	59925. 65	80059. 71	74854. 94	133591. 72
50	59948. 93	80038. 27	74900. 33	133510. 75
51	59972. 31	80020. 83	74945. 75	133429. 84
52	59995. 49	80003. 38	74991. 19	133340. 00
53	60018. 76	79985. 93	75036. 65	133268. 23
54	60042. 02	79968. 47	75081. 12	133187. 49
55	60065. 38	79951. 00	75127. 62	133106. 84
56	60088. 53	79933. 52	75173. 14	133026. 24
57	60111. 78	79916. 04	75218. 67	133045. 71
58	60135. 03	79898. 55	75264. 23	132865. 24
59	60158. 27	79881. 05	75309. 81	132784. 83
60	60181. 50	79863. 55	75355. 40	132704. 48

Canon Sinuum, Tangentium & Secantium.

37	Sinus	Tangens	Secans	
0	60181.50	79863.53	75355.40	132704.48
1	60204.73	79846.04	75401.02	132624.80
2	60227.95	79828.53	75446.66	132543.97
3	60251.17	79811.00	75492.32	132463.81
4	60274.39	79793.47	75537.99	132383.71
5	60297.60	79775.03	75583.69	132303.68
6	60320.80	79758.39	75629.41	132223.70
7	60344.00	79740.84	75675.14	132143.79
8	60367.19	79723.28	75720.90	132063.93
9	60390.38	79705.72	75766.68	131984.14
10	60413.56	79688.15	75812.48	131904.41
11	60436.74	79670.57	75858.29	131824.74
12	60459.91	79652.99	75904.13	131745.13
13	60483.08	79635.40	75949.99	131665.59
14	60506.24	79617.80	75995.87	131586.10
15	60529.40	79600.20	76041.77	131506.68
16	60552.55	79582.59	76087.69	131427.31
17	60575.70	79564.97	76133.63	131348.01
18	60598.84	79547.35	76179.59	131268.76
19	60621.98	79529.73	76225.57	131189.58
20	60645.11	79511.08	76271.17	131110.46
21	60668.23	79494.43	76317.59	131031.40
22	60691.35	79476.78	76363.63	130074.39
23	60714.47	79459.12	76409.69	130873.45
24	60737.58	79441.46	76455.77	130794.57
25	60760.69	79423.79	76501.88	130715.75
26	60783.79	79406.11	76548.00	130636.99
27	60806.89	79388.43	76594.14	130558.28
28	60829.98	79370.74	76640.31	130479.64
29	60853.06	79353.04	76686.49	130401.06
30	60876.14	79339.33	76732.70	130322.54
31	60899.22	79317.62	76778.93	130244.07
32	60921.29	79299.90	76825.17	130165.67
33	60945.35	79282.18	76871.44	130087.32
34	60968.43	79264.45	76917.73	130009.04
35	60991.47	79246.71	76964.04	129930.81
36	61014.52	79228.96	77010.37	129852.65
37	61037.56	79211.21	77056.72	129774.54
38	61060.60	79193.45	77103.09	129696.49
39	61083.63	79175.69	77149.48	129618.50
40	61106.66	79157.92	77195.89	129540.57
41	61129.68	79140.14	77242.33	129462.69
42	61152.70	79122.35	77288.79	129384.88
43	61175.72	79104.56	77335.26	129307.12
44	61198.73	79086.76	77381.75	129229.43
45	61221.73	79068.96	77428.27	129151.79
46	61244.73	79051.15	77474.81	129074.21
47	61267.72	79033.33	77521.37	128996.69
48	61290.71	7915.50	77567.95	128919.21
49	61313.69	78997.67	77614.55	128841.82
50	61336.66	78979.83	77661.17	128764.47
51	61359.63	78961.98	77707.82	128687.18
52	61382.60	78944.13	77754.48	128609.95
53	61405.56	78926.27	77801.17	128531.77
54	61428.52	78908.41	77847.88	128455.66
55	61451.47	78890.54	77894.60	128378.60
56	61474.42	78872.66	77941.35	128301.60
57	61497.36	78854.77	77988.12	128224.66
58	61520.29	78836.88	78034.92	128147.76
59	61543.22	78818.98	78081.73	128070.93
60	61566.15	78801.07	78128.58	127994.16

Canon Sinuum, Tangentium & Secantium.

38	Sinus	Tangens	Secans	
0	61566.15	78801.07	78128.56	127994.16
1	61589.07	78783.16	78175.42	127917.45
2	61611.98	78765.24	78213.29	127840.79
3	61634.89	78747.33	78269.19	127764.19
4	61657.79	78729.39	78316.11	127687.64
5	61680.69	78711.45	78363.05	127611.16
6	61703.59	78693.50	78410.02	127534.73
7	61726.48	78675.55	78457.00	127458.36
8	61749.36	78657.59	78504.00	127382.04
9	61772.24	78639.62	78551.03	127305.78
10	61795.11	78621.65	78598.08	127229.57
11	61817.98	78603.67	78647.15	127153.42
12	61840.84	78585.69	78692.24	127077.33
13	61863.70	78567.70	78739.35	127001.30
14	61886.55	78549.70	78786.49	126925.31
15	61909.40	78531.69	78833.64	126749.39
16	61932.24	78513.68	78880.82	126773.53
17	61955.07	78495.66	78928.02	126697.72
18	61977.90	78477.64	78975.24	126621.96
19	62000.73	78459.61	79022.48	126546.26
20	62023.55	78441.57	79069.75	126470.62
21	62046.36	78423.53	79117.03	126395.03
22	62069.17	78405.47	79164.34	126319.50
23	62091.98	78387.41	79211.67	126244.02
24	62114.78	78369.35	79259.02	126168.60
25	62137.57	78351.28	79306.40	126093.23
26	62160.36	78333.20	79353.79	126017.92
27	62183.14	78315.11	79401.21	125942.67
28	62205.93	78297.02	79448.65	125867.47
29	62228.69	78278.92	79496.11	125792.32
30	62251.46	78260.82	79543.59	125717.23
31	62274.21	78242.71	79591.10	125642.19
32	62296.98	78224.59	79638.62	125567.21
33	62319.73	78206.46	79686.17	125492.29
34	62342.48	78188.33	79733.74	125417.43
35	62365.23	78170.19	79781.34	125342.60
36	62387.96	78152.05	79828.95	125267.84
37	62410.69	78133.90	79876.59	125193.13
38	62433.42	78115.74	79924.25	125118.48
39	62456.14	78097.57	79971.93	125043.88
40	62478.85	78079.40	80019.63	124969.33
41	62501.56	78061.22	80067.36	124894.84
42	62524.26	78043.04	80115.11	124810.40
43	62546.96	78024.85	80162.88	124746.02
44	62569.66	78006.65	80210.67	124671.69
45	62592.35	77983.45	80358.48	124597.42
46	62615.03	77970.24	80306.31	124523.20
47	62637.78	77952.02	80354.18	124449.03
48	62660.38	77933.80	80402.06	124374.92
49	62683.05	77915.57	80449.97	124300.86
50	62705.71	77897.33	80497.90	124226.85
51	62728.37	77879.08	80545.85	124152.90
52	62751.02	77860.83	80593.82	124079.00
53	62773.66	77842.57	80641.81	124005.15
54	62796.30	77824.31	80689.83	123931.36
55	62818.94	77806.04	80737.87	123857.62
56	62841.57	77787.77	80785.93	123783.98
57	62864.20	77769.49	80834.01	123710.30
58	62886.83	77751.20	80882.12	123636.72
59	62909.43	77733.90	80930.25	123563.19
60	62932.04	77714.60	80978.40	123489.72

Canon Sinuum, Tangentium & Secantium.

39	Sinus	Tangens	Secans	
0	61932.04	77714.60	80978.40	123489.72
1	61934.64	77696.29	81056.58	123416.29
2	62977.24	77677.97	81074.78	123342.92
3	63099.83	77659.65	81123.00	123269.61
4	63021.42	77641.32	81171.24	123196.34
5	63045.00	77621.98	81219.51	123123.13
6	63067.58	77604.64	81267.80	123049.97
7	63090.15	77586.29	81316.11	122976.87
8	63112.72	77567.94	81364.44	122903.81
9	63135.28	77549.58	81412.80	122830.81
10	63157.84	77531.21	81461.18	122757.86
11	63180.39	77512.83	81509.58	122684.96
12	63202.93	77494.45	81558.01	122612.11
13	63225.47	77476.06	81606.46	122539.32
14	63248.00	77457.67	81654.93	122466.58
15	63270.53	77439.27	81703.43	122393.89
16	63293.05	77420.86	81751.95	122321.35
17	63315.57	77402.44	81800.49	122248.66
18	63338.08	77384.02	81849.05	122176.13
19	63360.59	77365.59	81897.64	122103.64
20	63383.09	77347.16	81946.25	122031.21
21	63405.59	77328.72	81994.88	121958.83
22	63428.08	77310.27	82043.54	121886.50
23	63450.57	77291.82	82092.82	121814.28
24	63473.05	77273.36	82140.93	121741.09
25	63495.53	77254.89	82189.65	121669.83
26	63518.00	77236.42	82238.40	121597.69
27	63540.46	77217.94	82287.18	121525.62
28	63564.92	77199.45	82335.97	121453.39
29	63585.37	77180.96	82384.79	121381.62
30	63607.82	77161.46	82433.64	121309.70
31	63630.26	77143.95	82481.51	121237.83
32	63652.70	77125.44	82531.40	121166.01
33	63675.13	77106.91	82580.31	121094.24
34	63697.56	77088.39	82629.25	121023.52
35	63719.98	77069.86	82678.21	120950.85
36	63742.40	77051.32	82727.19	120879.23
37	63764.81	77032.78	82776.20	120807.67
38	63787.21	77014.23	82825.23	120736.15
39	63809.61	76995.67	82874.29	120664.68
40	63832.01	76977.10	82923.37	120593.27
41	63854.40	76958.53	82972.47	120521.90
42	63876.78	76939.95	83021.60	120450.58
43	63899.16	76921.37	83070.75	120379.31
44	63911.53	76902.78	83119.98	120308.10
45	63943.90	76884.18	83169.12	120236.93
46	63966.26	76865.58	83218.34	120165.81
47	63988.62	76846.97	83267.59	120094.75
48	64010.97	76828.35	83316.86	120023.73
49	64033.33	76809.73	83366.15	119952.96
50	64055.66	76791.10	83415.47	119881.84
51	64077.99	76772.46	83464.81	119810.97
52	64100.32	76753.83	83514.18	119740.15
53	64122.64	76735.17	83563.57	119669.38
54	64144.96	76616.51	83612.98	119598.66
55	64167.27	76697.85	83662.41	119537.99
56	64189.58	76679.18	83711.88	119479.36
57	64211.88	76660.51	83761.36	119386.79
58	64234.18	76641.83	83810.87	119316.26
59	64256.47	76623.14	83860.40	119245.79
60	64278.76	76604.44	83909.96	119175.36

Canon Sinuum, Tangentium & Secantium.

40	Sinus	Tangens	Secans	
0	64278.76	76604.44	83909.96	119175.36
1	64301.04	76585.74	83959.54	119104.98
2	64323.32	76567.03	84009.15	119034.65
3	64345.59	76548.32	84058.78	118064.37
4	64367.87	76529.60	84108.44	118894.14
5	64390.11	76510.87	84158.12	118823.95
6	64412.36	76492.14	84207.82	118753.82
7	64434.61	76473.40	84257.55	118683.73
8	64456.85	76454.65	84307.30	118613.69
9	64479.09	76435.90	84357.08	118543.70
10	64501.34	76417.14	84406.88	118473.76
11	64523.55	76398.37	84456.70	118403.87
12	64545.77	76379.60	84506.55	118344.02
13	64567.98	76360.82	84556.43	118306.22
14	64590.19	76342.04	84606.33	118194.47
15	64612.40	76323.25	84656.25	118124.77
16	64634.60	76304.45	84706.20	118055.12
17	64656.79	76285.64	84756.17	117985.51
18	64678.98	76266.83	84806.17	117915.95
19	64701.16	76248.01	84856.19	117846.44
20	64723.34	76229.19	84906.34	117776.98
21	64745.51	76210.36	84956.31	117707.56
22	64767.67	76191.52	85006.40	117638.20
23	64789.83	76172.68	85056.52	117508.88
24	64811.99	76153.83	85106.67	117499.60
25	64834.14	76134.97	85156.84	117430.38
26	64856.28	76116.11	85207.04	117361.20
27	64878.42	76097.24	85257.26	117322.07
28	64900.55	76078.37	85307.50	117222.98
29	64922.68	76059.49	85357.77	117153.95
30	64944.80	76040.60	85408.07	117084.96
31	64966.92	76021.70	85458.39	117016.01
32	64989.03	76002.80	85508.73	116947.12
33	65011.14	75983.89	85559.10	116878.29
34	65033.24	75964.98	85609.50	116809.47
35	65055.33	75946.06	85659.92	116740.71
36	65077.42	75927.13	85710.37	116672.00
37	65099.50	75908.20	85760.84	116603.34
38	65121.58	75889.26	85811.33	116534.72
39	65143.66	75870.31	85861.85	116466.15
40	65165.72	75851.36	85912.40	116397.63
41	65187.78	75832.40	85962.97	116329.16
42	65209.84	75813.43	86013.57	116260.73
43	65231.89	75794.46	86064.19	116192.34
44	65253.94	75775.48	86114.84	116124.00
45	65275.98	75756.55	86165.51	116055.71
46	65298.01	75737.51	86216.21	115987.47
47	65320.04	75718.51	86266.93	115919.27
48	65342.06	75699.50	86317.68	115851.11
49	65364.08	75680.49	86368.46	115783.01
50	65386.09	75661.47	86419.26	115714.95
51	65408.10	75642.45	86470.09	115646.93
52	65430.10	75623.43	86520.94	115578.96
53	65452.09	75604.39	86571.81	115511.04
54	65474.08	75585.35	86622.71	115443.16
55	65496.06	75566.30	86673.64	115375.32
56	65518.04	75547.24	86724.60	115307.54
57	65540.01	75528.18	86775.58	115239.79
58	65561.98	75509.11	86826.59	115178.20
59	65583.94	75490.04	86877.62	115104.45
60	65605.90	75470.96	86928.68	115036.84

Canon Sinuum, Tangentium & Secantium.

41	Sinus	Tangens	Secans	
0	65605.90	75470.96	86928.68	115036.84
1	65627.85	75451.87	86976.76	114969.28
2	65649.80	75432.78	87030.87	114901.76
3	65671.74	75413.68	87082.00	114834.29
4	65693.67	75394.57	87133.16	114766.87
5	65715.60	75375.46	87184.35	114699.49
6	65737.53	75356.34	87235.16	114631.15
7	65759.44	75337.21	87286.80	114564.86
8	65781.35	75318.08	87338.06	114497.61
9	65803.26	75298.94	87389.35	114430.41
10	65825.16	75279.80	87440.67	114362.26
11	65847.06	75260.65	87492.01	114296.13
12	65868.95	75241.49	87543.38	114229.08
13	65890.83	75222.33	87594.78	114161.06
14	65912.71	75203.16	87646.20	114095.08
15	65934.58	75183.98	87697.65	114038.15
16	65956.45	75164.80	87749.12	113961.26
17	65978.31	75145.61	87800.62	113904.41
18	66000.17	75126.41	87852.15	113827.61
19	66021.03	75107.21	87903.70	113760.85
20	66043.86	75088.00	87955.28	113694.14
21	66065.70	75068.79	88006.89	113627.47
22	66087.53	75049.57	88058.53	113560.83
23	66109.36	75030.34	88101.18	113494.27
24	66131.18	75011.11	88151.86	113437.73
25	66153.00	74991.87	88213.57	113361.24
26	66174.81	74972.61	88265.31	113294.79
27	66196.62	74953.37	88317.07	113238.39
28	66218.42	74934.11	88368.86	113164.03
29	66240.22	74914.84	88420.68	113095.71
30	66262.01	74895.57	88472.53	113029.44
31	66283.79	74876.39	88524.40	112963.21
32	66305.57	74857.01	88576.30	112897.02
33	66327.34	74837.78	88628.23	112830.88
34	66349.11	74818.43	88680.17	112764.78
35	66370.87	74799.12	88732.15	112698.72
36	66392.62	74779.81	88784.16	112632.73
37	66414.37	74760.49	88836.20	112566.74
38	66436.11	74741.17	88888.26	112500.81
39	66457.85	74721.84	88940.34	112434.93
40	66479.59	74702.51	88992.45	112369.09
41	66501.32	74683.17	89044.59	112103.29
42	66523.04	74663.82	89096.75	112237.54
43	66544.75	74644.46	89148.94	112171.83
44	66566.46	74625.10	89201.16	112106.16
45	66588.17	74605.74	89253.41	112040.53
46	66609.87	74586.37	89305.69	111974.95
47	66631.56	74566.99	89357.99	111909.41
48	66653.25	74547.60	89410.32	111841.91
49	66674.93	74528.21	89462.68	111778.46
50	66696.61	74508.81	89515.06	111713.05
51	66718.28	74489.40	89567.47	111647.68
52	66739.94	74469.99	89619.91	111584.35
53	66761.60	74450.57	89671.38	111517.06
54	66783.26	74431.15	89724.87	111451.82
55	66804.91	74411.72	89777.39	111386.62
56	66826.55	74392.39	89839.94	111321.46
57	66848.18	74372.85	89882.52	111256.35
58	66869.81	74353.40	89935.12	111191.27
59	66891.44	74333.94	89987.75	111126.24
60	66913.06	74314.48	90040.41	111061.25

Canon Sinuum, Tangentium & Secantium.

42	Sinus	Tangens	Secans	
0	66933.06	74314.48	90040.41	111061.23
1	66934.67	74395.01	90093.09	110996.30
2	66936.28	74275.54	90145.80	110931.48
3	66977.88	74356.06	90198.54	110866.53
4	66999.48	74236.57	90251.31	110801.71
5	67021.07	74179.08	90304.11	110736.93
6	67042.66	74197.58	90356.94	110672.19
7	67064.24	74178.08	90409.79	110637.50
8	67085.82	74158.57	90462.67	110542.84
9	67107.39	74139.05	90515.58	110478.23
10	67128.97	74119.53	90568.51	110413.65
11	67150.51	74100.80	90621.47	110349.12
12	67172.06	74080.46	90674.46	110284.63
13	67193.61	74060.98	90727.48	110220.19
14	67215.15	74041.37	90780.53	110155.78
15	67236.68	74021.81	90833.60	110091.41
16	67258.21	74002.25	90886.78	110039.09
17	67279.73	73982.68	90939.84	109962.81
18	67301.25	73963.11	90993.00	109898.56
19	67322.76	73943.53	91046.19	109834.36
20	67344.27	73923.94	91099.41	109770.30
21	67365.77	73904.35	91152.65	109706.08
22	67387.27	73884.75	91205.92	109648.01
23	67408.76	73865.15	91259.22	109577.97
24	67430.24	73845.54	91322.55	109513.97
25	67451.72	73825.92	91365.91	109450.02
26	67473.19	73806.29	91419.29	109386.10
27	67494.66	73786.66	91472.70	109322.23
28	67516.12	73767.02	91536.15	109258.40
29	67537.57	73747.38	91579.62	109194.60
30	67559.02	73727.73	91631.11	109130.85
31	67580.46	73708.08	91686.65	109067.14
32	67601.90	73688.42	91740.20	109003.47
33	67623.33	73668.79	91793.79	108939.83
34	67644.76	73649.07	91846.40	108876.24
35	67666.18	73629.32	91901.04	108812.69
36	67687.60	73609.71	91954.71	108749.18
37	67709.01	73590.02	92006.41	108685.71
38	67730.41	73570.32	92062.14	108622.38
39	67751.81	73550.61	92115.90	108558.89
40	67773.20	73530.90	92169.68	108495.54
41	67794.59	73511.18	92223.50	108432.23
42	67815.97	73491.46	92277.34	108368.96
43	67837.34	73471.73	92334.22	108305.73
44	67858.71	73451.99	92385.11	108242.54
45	67880.07	73432.25	92439.05	108179.39
46	67901.43	73412.50	92493.01	108116.28
47	67922.78	73392.75	92547.00	108053.21
48	67944.13	73372.99	92601.01	107990.18
49	67965.47	73353.22	92655.06	107927.18
50	67986.81	73333.45	92709.14	107864.23
51	68008.14	73313.67	92763.24	107801.32
52	68029.46	73293.88	92817.38	107738.44
53	68050.78	73274.09	92871.54	107675.61
54	68072.09	73254.29	92925.73	107622.82
55	68093.39	73234.48	92979.96	107550.06
56	68114.69	73214.67	93034.21	107487.34
57	68135.99	73194.85	93088.49	107424.67
58	68157.28	73175.03	93142.80	107368.08
59	68178.56	73155.20	93197.14	107299.43
60	68199.84	73135.37	93251.51	107236.87

Canon Sinuum, Tangentium & Secantium.

43	Sinus	Tangens	Secans				
0	68199.84	73135.37	93151.51	107236.87	136732.75	146629.93	60
1	68221.11	73115.58	93305.91	107174.35	136769.85	146582.20	59
2	68242.37	73095.68	93360.34	107111.87	136806.99	146536.52	58
3	68263.63	73075.83	93414.79	107049.43	136844.16	146490.88	57
4	68284.88	73055.97	93469.38	106987.02	136881.36	146445.29	56
5	68306.13	73036.10	93523.80	106924.66	136918.59	146399.73	55
6	68327.37	73016.23	93578.34	106861.33	136955.86	146354.32	54
7	68348.61	72996.35	93632.92	106800.04	136993.15	146308.75	53
8	68369.84	72976.46	93687.53	106737.79	137040.48	146263.31	52
9	68391.07	72956.57	93742.16	106675.58	137067.84	146227.93	51
10	68412.29	72936.67	93796.83	106613.41	137105.23	146172.57	50
11	68433.50	72916.77	93851.52	106551.28	137142.66	146127.26	49
12	68454.71	72896.86	93906.25	106489.18	137180.11	146081.98	48
13	68475.91	72876.94	93961.01	106427.13	137227.60	146036.75	47
14	68497.11	72857.02	94015.79	106365.11	137255.12	145997.36	46
15	68518.30	72837.09	94070.61	106303.13	137293.68	145946.41	45
16	68539.48	72817.16	94125.45	106241.19	137330.26	145901.30	44
17	68560.66	72797.23	94180.33	106179.29	137367.88	145846.23	43
18	68581.83	72777.27	94235.23	106117.42	137405.53	145811.20	42
19	68603.00	72757.32	94290.17	106055.60	137443.31	145766.21	41
20	68624.16	72737.36	94345.13	105993.81	137480.92	145721.27	40
21	68645.33	72717.40	94400.13	105932.06	137518.67	145676.30	39
22	68666.47	72697.43	94455.16	105870.34	137556.45	145631.49	38
23	68687.61	72677.45	94510.21	105808.67	137594.26	145586.66	37
24	68708.75	72657.47	94565.30	105747.03	137632.10	145541.87	36
25	68719.88	72637.48	94620.42	105685.44	137669.98	145497.12	35
26	68731.01	72617.48	94675.56	105623.88	137707.89	145452.41	34
27	68752.13	72597.48	94730.74	105562.35	137745.83	145407.74	33
28	68793.24	72577.47	94785.95	105500.87	137783.80	145363.11	32
29	68814.35	72557.46	94841.19	105439.42	137821.81	145318.52	31
30	68835.45	72537.44	94896.46	105378.01	137859.85	145273.97	30
31	68856.55	72517.41	94951.76	105316.64	137897.92	145229.46	29
32	68877.64	72497.38	95007.09	105255.31	137936.02	145184.98	28
33	68898.73	72477.34	95062.45	105194.01	137974.16	145140.55	27
34	68919.81	72457.29	95119.84	105132.75	138012.33	145096.16	26
35	68940.89	72437.24	95173.26	105071.53	138050.53	145051.81	25
36	68961.96	72417.18	95228.71	105010.34	138088.77	145007.49	24
37	68983.08	72397.12	95284.20	104949.20	138127.04	144963.82	23
38	69004.07	72377.05	95339.71	104888.09	138165.34	144918.98	22
39	69025.12	72356.98	95395.26	104827.02	138203.67	144874.78	21
40	69046.17	72336.90	95450.83	104765.98	138242.04	144830.63	20
41	69067.21	72316.81	95506.44	104704.98	138280.44	144786.51	19
42	69088.24	72296.71	95562.08	104644.02	138320.87	144743.43	18
43	69109.27	72276.61	95617.74	104583.10	138357.34	144698.39	17
44	69130.29	72256.51	95673.44	104523.21	138395.84	144654.39	16
45	69151.31	72236.40	95729.17	104461.36	138434.37	144610.43	15
46	69172.32	72216.28	95784.94	104400.55	138474.94	144566.51	14
47	69193.32	72196.15	95840.73	104339.77	138511.54	144522.61	13
48	69214.32	72176.02	95896.55	104279.04	138550.27	144478.78	12
49	69235.31	72155.88	95952.41	104218.33	138588.83	144434.79	11
50	69256.30	72135.74	96008.29	104157.67	138627.53	144391.20	10
51	69277.28	72115.59	96064.21	104097.04	138666.26	144347.48	9
52	69298.25	72095.44	96120.16	104036.45	138705.03	144303.79	8
53	69319.22	72075.28	96176.14	103975.89	138743.83	144260.13	7
54	69340.18	72055.11	96232.15	103915.37	138782.66	144216.52	6
55	69361.14	72034.94	96288.19	103854.89	138821.53	144172.95	5
56	69382.09	72014.76	96344.27	103794.45	138860.42	144129.41	4
57	69403.04	71994.57	96400.37	103734.04	138899.36	144085.91	3
58	69423.98	71974.38	96456.51	103673.67	128938.32	144042.46	2
59	69444.91	71954.18	96511.68	103613.33	138977.32	143999.04	1
60	69465.84	71933.98	96568.88	103553.03	139016.36	143955.65	0

Canon Siniuum, Tangentium & Secantium.

42	Sinus	Tangens	Secans				
0	66913.06	74314.48	90040.48	111061.25	134563.27	149447.65	60
1	66934.67	74195.01	90093.09	110996.30	134598.53	149399.40	59
2	66956.28	74275.54	90145.80	110931.40	134633.82	149351.18	58
3	66977.88	74256.06	90198.54	110866.53	134669.14	149303.02	57
4	66999.48	74236.57	90251.31	110801.91	134704.49	149254.88	56
5	67011.07	74217.08	90304.11	110736.93	134739.87	149206.80	55
6	67022.66	74197.58	90356.94	110672.19	134775.28	149158.75	54
7	67044.24	74178.08	90409.79	110607.50	134810.71	149110.76	53
8	67085.83	74158.57	90461.67	110542.84	134846.19	149062.80	52
9	67107.43	74139.05	90515.58	110478.23	134881.69	149014.89	51
10	67128.95	74119.53	90568.51	110413.65	134917.21	148967.03	50
11	67150.51	74100.00	90621.47	110349.12	134952.77	148919.20	49
12	67172.06	74080.46	90674.46	110284.63	134988.36	148817.63	48
13	67193.61	74060.92	90727.48	110220.10	135023.98	148843.69	47
14	67215.15	74041.37	90780.53	110159.78	135059.63	148775.99	46
15	67236.68	74021.81	90833.60	110091.41	135095.31	148728.34	45
16	67258.21	74002.25	90886.71	110027.09	135131.03	148680.73	44
17	67279.73	73982.68	90939.84	109962.81	135166.76	148633.17	43
18	67301.25	73963.11	90993.00	109898.56	135202.54	148585.65	42
19	67322.76	73943.53	91046.19	109834.36	135238.34	148538.17	41
20	67344.27	73923.94	91099.41	109770.20	135274.17	148495.73	40
21	67365.77	73904.35	91152.65	109706.08	135310.03	148443.34	39
22	67387.27	73884.75	91205.92	109648.01	135349.93	148395.99	38
23	67408.76	73865.15	91259.12	109577.97	135381.86	148348.68	37
24	67430.34	73845.54	91312.55	109513.97	135417.81	148301.42	36
25	67451.72	73825.92	91365.91	109450.01	135453.79	148254.20	35
26	67473.19	73806.29	91419.19	109386.10	135489.80	148207.02	34
27	67494.66	73786.66	91472.70	109322.13	135525.85	148159.88	33
28	67516.12	73767.02	91526.17	109258.40	135561.93	148112.78	32
29	67537.57	73747.38	91579.62	109194.60	135598.03	148065.73	31
30	67559.02	73727.73	91633.12	109130.85	135634.17	148018.72	30
31	67580.46	73708.08	91686.65	109067.14	135670.34	147971.76	29
32	67601.90	73688.42	91740.20	109003.47	135706.54	147924.83	28
33	67613.33	73668.75	91793.79	108939.83	135742.77	147877.95	27
34	67644.76	73649.07	91847.40	108876.14	135779.03	147831.11	26
35	67666.18	73629.39	91901.04	108812.69	135815.38	147784.31	25
36	67687.60	73609.71	91954.71	108749.18	135851.64	147737.55	24
37	67709.01	73590.02	92008.41	108685.71	135888.00	147660.84	23
38	67730.41	73570.32	92062.14	108622.28	135924.38	147644.17	22
39	67751.81	73550.61	92115.90	108558.89	135960.80	147597.54	21
40	67773.20	73530.90	92169.68	108495.54	135997.25	147550.95	20
41	67794.59	73511.18	92223.50	108432.23	136033.71	147504.40	19
42	67815.97	73491.46	92277.34	108368.96	136070.23	147457.90	18
43	67837.34	73471.73	92331.22	108305.73	136106.77	147411.44	17
44	67858.71	73451.99	92385.12	108242.54	136143.34	147365.01	16
45	67880.07	73432.25	92439.05	108179.39	136179.95	147318.64	15
46	67901.43	73412.50	92493.01	108126.18	136216.58	147271.30	14
47	67922.78	73392.75	92547.00	108053.21	136253.24	147226.00	13
48	67944.13	73372.99	92601.01	107990.18	136289.94	147179.75	12
49	67965.47	73353.22	92655.06	107927.18	136326.67	147133.33	11
50	67986.81	73333.45	92709.14	107864.23	136363.43	147087.96	10
51	68008.14	73313.67	92763.24	107802.32	136400.22	147041.23	9
52	68029.46	73293.88	92817.38	107738.44	136437.04	146995.14	8
53	68050.78	73274.09	92871.54	107675.61	136473.89	146949.10	7
54	68072.09	73254.20	92915.73	107622.82	136510.78	146903.09	6
55	68093.39	73234.48	92979.96	107550.06	136546.70	146857.13	5
56	68114.69	73214.67	93034.21	107487.34	136584.64	146811.80	4
57	68135.99	73194.85	93088.49	107424.67	136621.62	146765.33	3
58	68157.28	73175.03	93142.80	107362.03	136658.63	146719.48	2
59	68178.56	73155.20	93197.14	107309.43	136695.67	146673.69	1
60	68199.84	73135.37	93251.51	107236.87	136731.75	146627.92	0

Canon Sinuum, Tangentium & Secantium.

43	Sinus .	Tangens .	Secans .	
0	68199. 84	73135. 37	93151. 51	107236. 87
1	68211. 11	73115. 58	93305. 91	107174. 35
2	68242. 37	73095. 68	93360. 34	107111. 87
3	68263. 63	73075. 83	93414. 79	107049. 43
4	68284. 88	73055. 97	93469. 28	106987. 02
5	68306. 13	73036. 10	93523. 80	106924. 66
6	68327. 37	73016. 23	93578. 34	106862. 33
7	68348. 61	72996. 35	93632. 92	106800. 04
8	68369. 84	72976. 46	93687. 53	106737. 79
9	68391. 07	72956. 57	93742. 16	106675. 58
10	68412. 29	72936. 67	93796. 83	106613. 41
11	68433. 50	72916. 77	93851. 52	106551. 28
12	68454. 71	72896. 86	93906. 25	106489. 18
13	68475. 91	72876. 94	93961. 01	106427. 13
14	68497. 11	72857. 02	94015. 79	106365. 11
15	68518. 30	72837. 09	94070. 61	106303. 13
16	68539. 48	72817. 16	94125. 45	106241. 19
17	68560. 66	72797. 22	94180. 33	106179. 29
18	68581. 83	72777. 27	94235. 23	106117. 42
19	68603. 00	72757. 32	94290. 17	106055. 60
20	68624. 16	72737. 36	94345. 13	105993. 81
21	68645. 32	72717. 40	94400. 13	105932. 06
22	68666. 47	72697. 43	94455. 16	105870. 34
23	68687. 61	72677. 45	94510. 21	105808. 67
24	68708. 75	72657. 47	94565. 30	105747. 03
25	68719. 88	72637. 48	94620. 42	105685. 44
26	68751. 01	72619. 48	94675. 56	105623. 88
27	68772. 13	72597. 48	94730. 74	105562. 35
28	68793. 24	72577. 47	94785. 95	105500. 87
29	68814. 45	72557. 46	94841. 19	105439. 42
30	68835. 45	72537. 44	94896. 46	105378. 01
31	68856. 55	72517. 41	94951. 76	105316. 64
32	68877. 64	72497. 38	95007. 09	105255. 31
33	68898. 73	72477. 34	95062. 45	105194. 01
34	68919. 81	72457. 29	95119. 84	105132. 75
35	68940. 89	72437. 24	95173. 26	105071. 53
36	68961. 96	72417. 18	95238. 71	105010. 34
37	68983. 02	72397. 12	95284. 20	104949. 20
38	69004. 07	72377. 05	95339. 71	104888. 09
39	69025. 12	72356. 98	95395. 26	104827. 02
40	69046. 17	72336. 90	95450. 83	104765. 98
41	69067. 21	72316. 81	95506. 44	104704. 98
42	69088. 24	72296. 71	95562. 08	104644. 02
43	69109. 27	72276. 61	95617. 74	104583. 10
44	69130. 29	72256. 51	95673. 44	104523. 21
45	69151. 31	72236. 40	95729. 17	104461. 36
46	69172. 32	72216. 28	95784. 94	104400. 55
47	69193. 32	72196. 15	95840. 73	104339. 77
48	69214. 32	72176. 01	95896. 55	104279. 04
49	69235. 31	72155. 88	95952. 41	104218. 33
50	69256. 30	72135. 74	96008. 29	104157. 07
51	69277. 18	72115. 59	96064. 21	104097. 04
52	69298. 25	72095. 44	96120. 16	104036. 45
53	69319. 22	72075. 28	96176. 14	103975. 89
54	69340. 18	72055. 11	96232. 15	103915. 37
55	69361. 14	72034. 94	96288. 19	103854. 89
56	69382. 09	72014. 76	96344. 27	103794. 45
57	69403. 04	71994. 57	96400. 37	103734. 04
58	69423. 98	71974. 38	96456. 51	103673. 67
59	69444. 91	71954. 18	96512. 68	103613. 33
60	69465. 84	71933. 98	96568. 88	103553. 03

Canon Sinuum, Tangentium & Secantium.

44	Sinus	Tangens	Secans				
0	69465. 84	71933. 98	96968. 88	103553. 03	139016. 86	143955. 65	60
1	69466. 76	71913. 77	96635. 11	103492. 77	139055. 43	143912. 31	59
2	69507. 67	71893. 55	96681. 37	103481. 54	130094. 73	143869. 00	58
3	69528. 58	71873. 35	96737. 67	103374. 35	139133. 66	143855. 74	57
4	69549. 49	71853. 10	96794. 00	103312. 80	139172. 83	143782. 51	56
5	69570. 39	71832. 87	96850. 35	103252. 08	139212. 03	143739. 31	55
6	69591. 28	71812. 63	96906. 74	103191. 99	139251. 27	143696. 16	54
7	69612. 17	71792. 38	96963. 18	103131. 95	139190. 54	143653. 05	53
8	69633. 05	71772. 13	97019. 63	103071. 94	139329. 85	143609. 97	52
9	69653. 92	71751. 87	97076. 10	103011. 96	139369. 18	143566. 93	51
10	69674. 79	71731. 65	97132. 63	102952. 03	139408. 56	143523. 93	50
11	69695. 65	71711. 42	97189. 17	102892. 18	139447. 96	143480. 97	49
12	69716. 51	71691. 06	97245. 75	102832. 26	139487. 40	143438. 05	48
13	69737. 36	71670. 78	97302. 36	102772. 43	139526. 88	143395. 16	47
14	69758. 21	71650. 49	97359. 01	102712. 63	139566. 39	143352. 31	46
15	69779. 05	71630. 19	97415. 49	102652. 87	139605. 93	143309. 50	45
16	69799. 88	71609. 89	97472. 40	102593. 15	139645. 51	143266. 72	44
17	69820. 71	71589. 58	97539. 14	102533. 46	139683. 12	143243. 99	43
18	69841. 53	71569. 27	97585. 91	102473. 81	139724. 77	143181. 19	42
19	69862. 34	71548. 95	97642. 73	102414. 19	139764. 45	143138. 63	41
20	69883. 15	71528. 63	97699. 56	102374. 61	139804. 16	143096. 00	40
21	69903. 96	71508. 30	97756. 43	102329. 06	139843. 91	143053. 42	39
22	69924. 76	71487. 96	97813. 33	102235. 55	139883. 69	143010. 87	38
23	69945. 55	71467. 62	97870. 27	102176. 08	139923. 51	142968. 36	37
24	69966. 33	71447. 27	97927. 24	102116. 64	139963. 36	142925. 18	36
25	69987. 11	71426. 91	97984. 24	102057. 23	140003. 35	142883. 44	35
26	70007. 89	71406. 55	98041. 27	101997. 86	140043. 17	142841. 04	34
27	70018. 66	71386. 13	98098. 33	101928. 53	140083. 13	142798. 68	33
28	70049. 42	71365. 81	98155. 43	101879. 23	140123. 13	142716. 36	32
29	70070. 18	71345. 43	98212. 56	101819. 97	140163. 15	142714. 07	31
30	70090. 93	71325. 05	98269. 73	101760. 74	140203. 21	142671. 82	30
31	70111. 67	71304. 66	98326. 92	101701. 55	140243. 30	142659. 61	29
32	70132. 41	71284. 26	98384. 15	101642. 39	140283. 43	142587. 43	28
33	70153. 14	71263. 85	98441. 41	101583. 26	140323. 60	142545. 29	27
34	70173. 87	71243. 44	98498. 71	101514. 17	140363. 80	142503. 19	26
35	70194. 59	71223. 02	98556. 03	101465. 12	140404. 03	142461. 12	25
36	70215. 30	71202. 60	98613. 39	101406. 10	140444. 30	142419. 09	24
37	70236. 01	71181. 17	98670. 79	101347. 12	140484. 60	142377. 10	23
38	70256. 71	71161. 74	98728. 21	101288. 17	140524. 94	142335. 14	22
39	70277. 41	71141. 30	98785. 67	101229. 35	140565. 32	142293. 23	21
40	70298. 10	71120. 86	98843. 16	101170. 37	140605. 73	142251. 34	20
41	70318. 79	71100. 41	98900. 69	101111. 53	140646. 17	142209. 50	19
42	70339. 47	71079. 95	98958. 35	101052. 72	140686. 65	142167. 69	18
43	70360. 14	71059. 48	99015. 84	100993. 94	140727. 17	142125. 92	17
44	70380. 81	71039. 01	99073. 46	100935. 20	140767. 78	142084. 18	16
45	70401. 47	71018. 14	99131. 12	100876. 49	140808. 31	142042. 48	15
46	70422. 13	70998. 06	99188. 81	100817. 83	140848. 93	142000. 83	14
47	70442. 78	70977. 57	99246. 54	100759. 18	140889. 58	141959. 19	13
48	70463. 42	70957. 07	99304. 29	100700. 58	140920. 38	141917. 61	12
49	70484. 06	70936. 57	99362. 08	100642. 01	140971. 00	141876. 05	11
50	70504. 69	70916. 07	99419. 91	100583. 47	141011. 72	141834. 54	10
51	70525. 32	70895. 56	99477. 77	100524. 97	141052. 16	141793. 05	9
52	70545. 94	70875. 04	99535. 66	100466. 51	141093. 40	141751. 61	8
53	70566. 55	70854. 51	99593. 58	100408. 07	141134. 87	141710. 20	7
54	70587. 16	70833. 98	99651. 54	100349. 68	141175. 17	141668. 83	6
55	70607. 26	70813. 45	99709. 53	100291. 81	141216. 41	141627. 49	5
56	70628. 35	70792. 91	99767. 56	100232. 98	141257. 09	141586. 19	4
57	70648. 94	70772. 36	99825. 62	100174. 69	141298. 10	141544. 93	3
58	70669. 53	70751. 80	99883. 71	100116. 42	141339. 15	141593. 70	2
59	70690. 21	70731. 24	99941. 84	100058. 19	141380. 84	141462. 51	1
60	70710. 68	70710. 68	100000. 00	100000. 00	141421. 36	141421. 36	0

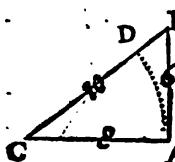
G E O M E T R I A
T R I A N G U L O R U M
L I B E R III.

De Rectilineorum Triangulorum Calculo.

CONSONANTIA Triangulorum compositus, facilem rectilineorum, Sphericorumque Triangulorum Calculum suppeditat.

Hec est tertia pars Triangulorum Geometria: Canonis Triangulorum compofitum usum ostendens, cumque duplicum. Primum in rectilineorum Triangulorum; Alterum in Sphericorum Triangulorum Calculo.

1. Triangulum rectilineum, est figura in planicie, tribus rectis lineis, quæ finibus suis se mutuo contingunt, conformata.

 Talis est figura A B C: est enim conformata in planicie, tribus rectis lineis A B, A C & B C, quæ finibus suis se mutuo contingunt.

3. Triangulum rectilineum, rectangulum est; aut obliquangulum.

4. Triangulum rectilineum rectangulum est, quod angulum habet rectum.

Tale est in figura superiori Triangulum A B C: habet enim angulum rectum ad A.

5. Anguli rectilinei amplitudinem determinat comprehensus ab eo arcus, qui super vertice anguli ipsius velut centro describitur.

Sic in figura premissa arcus A D, descriptus centro C, mensurat amplitudinem anguli B C A.

6. In Triangulo rectangulo quadratum basis est æquale quadratis laterum.

Basis Trianguli rectanguli vocatur recta linea qua angulum rectum subedit: reliqua vero rectum ambientes, latera dicuntur. Itaque in Triangulo superiori A B C, quadratum basis B C, est æquale quadratis laterum B A & C A: cuius ratio ex permutatio primi elementi, manifesta est.

P O P I S M A T A dvo.

Itaque lateribus trianguli rectanguli cognitis, invenitur & basis: collecta enim in unam summam laterum quadrata, componunt quadratum basis, cuius radix quadrata est ipsa basis quæ sita.

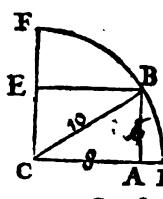
In exemplo fit latus A B 6; & quadratum ejus 36: A C 8, & quadratum 64: et B C 10. Iuncta enim summa quadrata 36 & 64, componunt quadratum 100: cuius radix quadrata est 10, pro B C basi quæ sita.

Data vero bafi cum latere alterutro, manifestatur & reliquum latus: subducto enim quadrato lateris dati, ex quadrato bafis, relinquatur quadratum reliqui lateris; cuius radix quadrata est mensura lateris quæ sita.

In exemplo premisso, deme quadratum lateris A C 64, ex quadrato bafis B C 100: relinquatur quadratum lateris A B 36; & radix ejus 6, pro ipso latere, ut supra. Item deme quadratum lateris A B 36, ex quadrato bafis B C 100: residuum erit quadratum lateris A C 64; & radix quadrata ejus 8, pro ipso latere postulato.

7. Si Trianguli rectanguli basis assumatur ut circuli radius, latera sinus recti sunt oppositorum angulorum.

Geometriæ Triangulorum Liber III.



Esto enim Triangulum rectangulum A B C , in quo B C basis assumatur ut circuli radius. Dico B A esse sinum rectum anguli B C A ; & A C sinum rectum anguli A B C . Recta enim B A est perpendicularis à termino arcus B in semidiametrum D A C . Itaque per 7 primi hujus . Sinus rectus est arcus D B vel anguli B C A per 5 hujus . Eadem ratione recta B E , est sinus rectus arcus E B , vel anguli B C E . Atque per 34 primi , A C aquatur B E ; & angulus A B C , aquatur angulo B C E : ergo A C , sinus est anguli A B C oppositi .

II O P I Σ M A T A quatuor.

Primo itaque data basi cum angulis inveniuntur latera . Nam ut radius se habet ad sinum anguli ; ita basis ad latus ipsi angulo oppositum .

Exempli gratia . Sit basis B C partium 10 , & angulus B C A partium 36 52' 11" , & A B C prioris complementi partium 53 7' 49" . Sinus autem A B 6000000 , & A C 8000000 , in ea mensura , in qua radius B C est 10000000 . Inveniuntur latera A B 6 , & A C 8 . Nam per 19 Septimi Euclidis ,
Vt B C 10000000 , ad A B 6000000 : Ita B C 10 , ad A B 6 . Item .
Vt B C 10000000 , ad A C 8000000 : Ita B C 10 , ad A C 8 .

Secundo , data basi cum latere alterutro , manifestantur anguli . Basis enim est ad latus datum : ut radius ad sinum anguli dicto lateri oppositi .

In eodem exemplo . datur B C 10 , & A B 6 : Inveniatur angulus A C B partium 36 52' 11" . Nam per 19 Septimi Euclidis .

Vt B C 10 , ad A B 6 : Ita B C 10000000 , ad A B 6000000 , sinum partium 36 52' 11" , competentem angulo A C B . Itaque A B C reliquus angulus , est partium 53 7' 49" : prioris scilicet complementum , ut ex 7 hujus , & 32 primi elementorum manifestum est .

Terzio , dato latere alterutro , cum angulis , investigatur latus reliquum . Sinus enim anguli dato latere oppositi , est ad sinum complementi sui : ut latus datum , ad latus reliquum .

Datur in eodem exemplo angulus A C B partium 36 52' 11" , & sinus ejus 6000000 : A B C partium 53 7' 49" , & sinus ejus 8000000 , cum latere A B 6 ; Dabitur A C reliquum latus 8 . Nam per 19 Septimi Euclidis ,

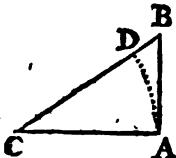
Vt A B 6000000 , ad A C 8000000 : Ita A B 6 , ad A C 8 .

Quarto , datis angulis , & latere alterutro , addiscitur basis : Sinus enim anguli dato latere oppositi , est ad radium : ut latus datum ad Basim .

Repetito & hic superiori exemplo . Datur A B 6 , & angulus ei oppositus B C A partium 36 52' 11" , cum sinus ejus 6000000 . Inveniatur basis B C partium 10 . Nam per 19 Septimi Euclidis ,

Vt A B 6000000 , ad B C 10000000 : Ita A B 6 , ad B C 10 .

8. Si Trianguli rectanguli latus alterutrum , ex acuto angulo , fiat circuli radius ; reliquum est ejusdem anguli Tangens .



B Esto rectangulum Triangulum A B C , cuius latus A C fiat circuli radius ex acuto angulo C . Dico A B , tangentem esse anguli A C B , vel arcus A D : est enim perpendicularis extremitate semidiametri A , in radius C D per arcus terminum D continuatur . Itaque per 14 Primi hujus , dicti anguli , vel arcus , Tangens est .

II O P I Σ M A T A du.

Primo igitur , dato latere alterutro cum angulis , inveniuntur reliquum latus . Radius enim est ad tangentem anguli latere quæsito oppositi : ut latus datum ad latus reliquum .

Exempli gratia datur latus A B 6 : & angulus A B C part . 53 7' 49" , itaque A B radius : erit A C Tangens anguli A B C ex Canone Tangentium 13333333 paulo plus ; & latus A C reliquum 8 . Nam per 19 Septimi Euclidis ,

Vt A B 10000000 , ad A C 13333333 paulo plus : Ita A B 6 , ad A C 8 , Omnia ut supra .

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Secundo, dato utroque latere, investigantur anguli. Nam ut latus alterum est ad latus reliquum; ita radius ad tangentem anguli reliquo lateri oppositi.

In exemplo detur latus A B 6: & reliquum latus A C 8. Invenietur angulus A B C lateri A C oppositi, partium 53° 7' 49". Nam per 19 Septimi Euclidis,

Vt A B 6, ad A C 8: Ita A B 10000000, ad A C 13333333 pauc plus, Tangentem anguli A C B, oppositi lateri A C; qui ex Tangentium Canone invenitur partium 53° 7' 49". Ergo reliquus angulus B C A est partium 36° 52' 11".

9. Si Trianguli rectanguli latus alterutrum est anguli tangens, basis est anguli ejusdem secans.

Repetita premisi Theorematis figura, Sit A B latus, Tangens anguli B C A. Dico Basim B D C esse ejusdem anguli Secantem: est enim ducta per terminum peripherie A D in Tangentem A B. Itaque per 19 primi bujus, Secans est peripherie A D, vel anguli B C A.

N O P I Z M A T A tria.

Primo ergo, dato latere alterutro, cum angulis, manifestatur basis. Radius enim est ad secantem anguli dati: ut latus eidem angulo adjacens ad Basin..

Exempli loco detur latus A C 8; & angulus B C A dato lateri adjacens, partium 36° 52' 11": secans ejus 12500000, erit Basis B C 10. Nam per 19 Septimi Euclidis,

Vt A C 10000000, ad B C 12500000: Ita A C 8, ad B C 10.

Secundo, dato latere alterutro & Basi, exquiruntur anguli. Nam ut latus alterutrum ad Basim: ita radius est ad secantem anguli lateri dato adjacentis.

In exemplo eodem, detur latus A C 8, & basi B C 10: erit angulus B C A partium 36° 52' 11". Nam per 19 Septimi Euclidis,

Vt A C 8, ad B C 10: Ita A C 10000000, ad B C 12500000, Secantem anguli B C A, lateri dato A C adjacentem. Inveniturque ex Canone secantium partium 36° 52' 11": ergo reliquus A B C, est partium 53° 7' 49".

Tertio datis angulis & Basi, inveniuntur latera. Nam secans anguli dati est ad radium: ut basis ad latus dato angulo adjacens.

Sit iterum exempli loco angulus B C A partium 36° 52' 11", & Secans ejus ex Canone Secantium 12500000: Basi B C 10; erit A C latus angulo dato adjacens 8. Nam per 19 Septimi Euclidis,

Vt B C 12500000, ad A C 10000000: Ita B C 10, ad A C 8.

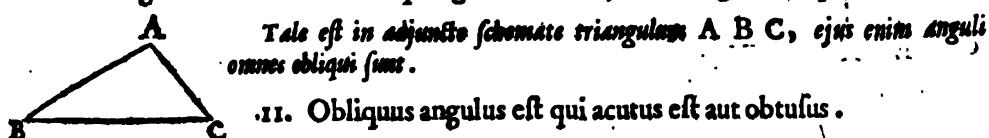
Rursus detur Secans anguli A B C partium 53° 7' 49". 16666666: & basi B C 10; erit A B 6. Nam per 19 Septimi Euclidis,

Vt B C 16666666, ad A B 10000000: Ita B C 10, ad A B 6.

Et sic Triangulorum Rectangulorum Calculum absolvimus. Sequitur

Obliquangulorum Triangulorum Calculus.

10. Triangulum rectilineum obliquangulum est, cujus tres anguli obliqui sunt.



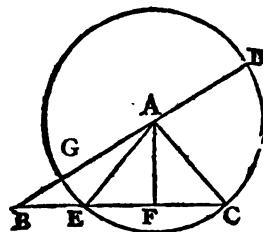
Tale est in adjuncto schemate triangulare A B C, ejus enim anguli omnes obliqui sunt.

11. Obliquus angulus est qui acutus est aut obtusus.

12. Acutus angulus est qui recto minor est: obtusus qui recto major.

Ita in superiori figura angulus ad B & C est acutus, est enim interque recto minor: angulus vero ad A obtusus est, quia recto major est.

13. Si trianguli obliquanguli latus minus fiat circuli radius, & ex ejus angulari punto describatur circulus, basi & latus majus secans; erit basis ad summam laterum, ut segmentum lateris ad segmentum basis.



Basis trianguli obliquanguli vocatur latus majus: vel, si aequicrurum sit, alterutrum crurum pro basi assumpsum. Sit ergo ABC Triangulum obliquangulum, cuius latus minus AC, basis BC: factio autem AC radio, ex A puncto angulari, describatur circuli peripheria, secans basim in E, reliquum latus in G. Dico basim BC, esse ad summam laterum BA & AC vel AD (AC enim & AD radius sum, & prouide aequales per 15 Definitionem primi elementorum) ut BG lateris segmentum, ad BE segmentum Basis. Resta enim BD & BC à B punto extra circulum producta secant circulum in G & E. Itaque per 36 Tertii elementorum, ut BC ad BD: Ita BG ad BE, quod erat demonstrandum.

H O P I S M A.

Itaque tribus obliquanguli Trianguli lateribus datis, inveniuntur tres anguli. Nam ut basis trianguli ad summam laterum, ita laterum differentia ad basis segmentum: sed ut basis segmentum cum semisse residui, est ad latus majus; ita radius ad secantem anguli lateri minori adjacentis. Item ut semissis residui, est ad latus minus: ita radius ad secantem anguli lateri minori adjacentis. Dantur ergo duo anguli: quibus ex semicirculo subductis, relinquitur tertius basi oppositus.

Retenta precedentis Trianguli figura sit basis BC 28. Latera vero BA 25, AC 17: & eorum summa BD 42; Differentia BG 8: erit BE segmentum Basis 12. Nam,

Ita BC 28, ad BD 42: Ita BG 8, ad BE 12.

Subductum vero segmentum BE 12, ex basi BC 28: relinquit EC 16; cuius semissis EF vel FC est 8 (Perpendicularis enim AF bisecat EC. per tertiam tertii elementorum) hinc dantur anguli ad B & C per 8 hujus. Nam BE est 12, & EF 8: ergo tota BF 20. Basis autem BA Trianguli rectanguli BFA est 25. Itaque

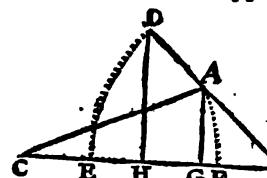
Ita BF 20, ad BA 25: Ita BF 10000000, ad BA 12500000, secantem anguli ad B partium 36 52' 11". Rursus in Triangulo AFC rectangulo dasur latus FC 8, & basis AC 17: ergo

Ita FC 8, ad AC 17: Ita FC 10000000, ad AC 21250000, secantem anguli ad C partium 61 55' 39".

Iam cum anguli ad C & B noti sint, non potest latere reliquus ad A: est enim residuus duorum ad semicirculum, per 32 primi elementorum. Dempto igitur utroque ex semicirculi partibus 180: relinquitur ipse angulus quaesitus, partium 81 12' 10".

Atque ita ex tribus obliquanguli Trianguli lateribus datis, tres anguli inventi sunt: quod erat ostendendum.

14. In Triangulo obliquangulo proportio lateris ad latus est: ut proportio sinus recti anguli alteri lateri oppositi, ad sinum rectum anguli reliquo lateri oppositi; & contra.



Sit Triangulum ABC obliquangulum, laterum inequalium (nam si latera equalia sint, anguli oppositi per 5 primi element. aquantur: itaque & sinus eorum aequales sunt per 29 tertii Element.) Dico BA latus esse, ad AC latus: ut sinus rectus anguli ACB, ad sinum rectum anguli ABC. Continuetur ensem BA latus in D, ut aequaliter sit lateri BAC: descriptisque peripheria DE & AF aequalibus radius CA & BD, dimittantur ex D & A arcum terminis perpendicularares in basim BC; sintque DH & AG sinus recti scilicet angulorum C & B, vel arcum AF & DH per 7 primi hujus. Erit per 4 sexti elementorum, ut BA ad BD, hoc est AC: Ita AG sinus rectus anguli oppositi, ad DH sinum rectum anguli oppositi: quod erat demonstrandum. Conversa hujus Theorematis eodem modo demonstratur. Nam quia BA latus unum est ad BD latus alterum: ac AG sinus anguli C oppositi, ad DH sinus anguli oppositi; est etiam, per elementum citatum, AG sinus anguli C, ad DH sinum

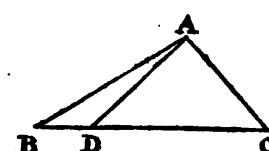
Geometriæ Triangulorum Liber III.

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anguli B; ut oppositum latus A B, ad oppositum latus D H, id est, B C. Quod etiam demonstrandum erat. Observa autem hoc Theorema verum esse non modo in omnibus rectilineis Triangulis, sed & Sphericis, quemadmodum suo loco demonstrabisur.

ΠΟΡΙΣΜΑΤΑ duo.

Itaque datis duobus obliquanguli Trianguli lateribus, & angulo non ab his comprehenso obtuso (aut si acuto data anguli specie alteri lateri oppositi) anguli reliqui, & latus tertium invenitur. Nam ut latus alterutrum dato angulo oppositum est ad sinum anguli dati: ita latus alterum, ad sinum anguli oppositi. Dantur ergo duo anguli; quibus ex semicirculo ablatis, relinquitur tertius. quare ut si quis anguli alterutrum poterit ad alterutrum latus oppositum; ita sinus anguli tertii, ad latus tertium.



Detur in Triangulo A B C obliquangulo, latus A B 25; A C 17: & angulus A B C non ab iis comprehensus acutus partium 36° 52' 11"; cum acuta specie anguli ad C ignoti. Invenietur ipse angulus ad C partium 61° 55' 39". Nam per 19 Septimi Euclidis,

Vt latus A C 17, ad sinum anguli A B C 600000: Ita A B latus 25, ad sinum anguli A C B 8823529.

Cujus arcus è sinuum Canone datur partium 61° 55' 39", quia species anguli acuta est: nam si obtusa esset, angulus existentes partium 118° 4' 21". Quod ut manifestum fuit, ducatur ex A recta A D in basim B C, aequalis A C: era A D C Triangulum equilaterum, & angulus A D C per 5 primi element. aequalis angulo A C D; exterior autem A D B per 13 ejusdem, aut reliqua ad semicirculum. Quare ut latus B A subtrahit duplicum angulum, A D B obtusum, & A C B acutum: Ita etiam sinus invenitus, per 7 primi huius est duarum peripheriarum, minoris circuli quadrante, & reliqua ad semiperipheriam. Patet igitur definitam esse anguli speciem dato angulo acuto existente. Alia vero est ratio, cum angulus obtusus datur: nam tunc manifestum est, reliquos Trianguli angulos acutos esse. Duo enim obtusi anguli in Triangulo plato esse nequeunt, cum omnes Trianguli anguli per 32 primi element. equeales sint duobus rectis. Itaque species anguli tunc per se data est, nempe acuta.

Porro cum in Triangulo A B C duo anguli recti sunt, A B C & A C B, non potest latere tertius B A C: est enim per 32 primi elementa, residuum duorum differens ad semicirculum, partium scilicet 81° 12' 10". Itaque tertium latus inde invenies. Nam

Vt sinus anguli A B C 600000, ad latus A C 17: Ita sinus anguli B A C 9882353, ad latus B C 28. vel,

Vt sinus anguli A C B 8823529, ad latus A B 25: Ita sinus anguli B A C 9882353, ad latus B C 28.

Secundo, datis duobus Trianguli obliquanguli angulis, & uno latere, manifestatur angulus tertius, cum reliquis lateribus. Subductis enim duobus angulis datis ex semicirculo, relinquitur tertius. quare ut se habet sinus anguli lateri dato oppositi ad latus datum: ita etiam reliquorum angulorum sinus ad latera opposita.

Sit & hic Triangulum obliquangulum A B C, cuius duo anguli A B C 36° 52' 11": & A C B 61° 55' 39 dentur; cum latere B C 28. Invenietur reliquus angulus B A C, cum lateribus B A & A C. Demptis enim angulis datis ex semicirculo, relinquatur angulus tertius B A C, partium 81° 12' 10". Itaque per 19 Septimi Euclidis,

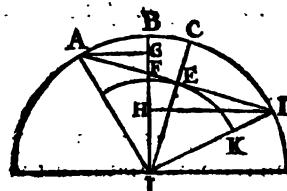
Vt sinus anguli B A C 9882352, ad latus B C 28: Ita sinus anguli A B C 6000000, ad latus A C 17. Item

Vt sinus anguli B A C 9882352, ad latus B C 28: Ita sinus anguli A C B 8823529, ad latus A B 25. vel

Vt sinus anguli A B C 6000000, ad latus A C 17: Ita sinus anguli A C B 8823529, ad latus A B 25.

15. Si angulorum duorum summa detur, quorum sinuum ratio inter se constet, ipsi etiam anguli secernuntur. Nam ut semissis summæ sinuum rationis, ad differentiam semissis, & termini rationis sinuum alterutrius est: ita tangens summæ angulorum, ad tangentem

gentem anguli; quo minor quæsitus ab anguloru^m summa semissæ deficit; major eam superat.

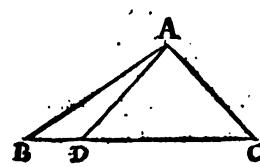


Detur in adjecto diagrammate, summa angulorum AIB & BID part. 40: cum ratione sinuum ut AG ad DH (vel per 4 sexti elementorum ut AF ad DF) ut 4 ad 7. Dico utrumque angulum AIB & BID signatim datum iri. Egrediatur enim ex I recta, biseccans AD, rationis sinuum summam datam in E: erit AE $\frac{5}{2}$, & angulus AIE partium 20; equalis angulo DIE; FE vero (differentia termini minoris AF 4, & AE $\frac{5}{2}$ vel ED $\frac{5}{2}$) & FD termini majoris 7) $\frac{1}{2}$. Fiat quoque IE radius, ut DE tangens sit anguli DIE, vel arcus KE partium 20: hinc enim dabitur tangens EIF vel BIC subtendens. Nam per 19 septimi Euclidis VI DE $\frac{5}{2}$ ad DE tangentem ang. DIE 3639702: Ita FE $\frac{1}{2}$, ad FE tang. anguli EIF, 992646.

Cujus arcus è Tangentium Canone datur partium 5 40' 8" fere. Atqui hoc angulo major est angulus BID major: minor vero angulus AIB minor. Ergo BID angulus est partium 25 40' 8" fere: AIB partium 14 19' 52": Quod erat demonstrandum.

H O P I S M A.

Itaque duobus obliquanguli Trianguli lateribus datis, & angulo ab iis comprehenso, inveniuntur anguli reliqui, & latus tertium. Nam ut semissæ summae laterum datorum, ad differentiam summae semissæ, & lateris alterutrius: Sic tangens semissæ residui anguli ad semicirculum, ad Tangentem anguli, quo angulus minori lateri oppositus eadem semissæ minor, majori major est. Dantur ergo tres anguli. Quare, ut sinus alterutrius anguli, ad latus oppositum: ita sinus anguli quæsito oppositi, ad latus quæsitum.



Retenta superioris Trianguli figura, sit latus AB 25; BC 28: & angulus ABC partium 36 52' 11". Invenientur reliqui anguli BAC, & ACB cum tercio latere AC. Nam per 32 primi elementorum, ex angulo B noto, datur summa angulorum BAC & ACB, partium 143 7' 48": residuum scilicet anguli dati ad semicirculum: item ex lateribus notis, datur ratio sinuum angulorum oppositorum per 13 hujus. Itaque cum angulorum duorum summa detur, cum ratione sinuum etiam eterque signatim definitur. Nam

Vt semissæ summa laterum 26 $\frac{1}{2}$. } ad differentiam summae semissæ & lateris alterutrius $\frac{1}{2}$:

Sic Tangens semissæ residui anguli, ad semicirculum partium 71 33' 54", sci- } ad Tangentem 1698112, anguli partium 9 38' 15", quo angulus A C B minori lateri oppositus semissæ residui anguli ad semicirculum minor est: reliquis B A C majori lateri oppositus major est. Itaque ACB est part. 61 55' 39", BAC 81 11' 9". ut supra.

Latus AC ex premiso Theoremate invenitur 17. Nam

Vt sinus anguli BAC 9882352 ad BC oppositum latus 28: Ita sinus anguli ABC 6000000, ad AC latus oppositum 17.

Et sic calculum rectilineorum Triangulorum exposuimus, cujus usus est in omni magnitudinum genere dimiendo. Supereft tantum ut in eo Mathematum studiosus sedulo se exerceat. Theorematum enim sunt pro inventione cuiusvis quarti in Triangulo rectilineo dato tribus, idque per 19 septimi Euclidis, .i. regulam proportionum.

Etsi vero superior doctrina tam clare proposita sit, ut ulterius explicari non sit opus: quo tamen promptior & expeditior sit ejus usus, subjungimus sequentem diatyposim, in qua tanquam in tabula doctrina superioris summam exhibemus.

I N T R I A N G U L O R E C T A N G U L O

inveniuntur.

L A T E R A

Ex basi & angulis, per 7 bujus.

Vt radius, ad sinum anguli queſito *I* *II* *III* *IV*
lateri oppofiti } Ita Bafis, ad latus queſitum:
vel per 9 bujus.

Vt secans anguli queſito *I* *II* *III* *IV*
lateri adjacens } ad Radium } Ita basiſ, ad latus queſitum.

Ex angulis & latere alterutro, per 7 bujus.

Vt sinus anguli dato *I* *II* *III* *IV*
lateri oppofiti } ad sinus comple- } Ita datum latus, ad latus re-
menti ſui } menti ſui } liquum.
vel per 8 bujus.

Vt radius ad Tangentem anguli dato *I* *II* *III* *IV*
lateri oppofiti } Ita datum latus, ad latus re-
liquum.

Ex basi & latere alterutro, per 6 bujus.

*Minus quadratum lateri noti ex quadrato basi, relinquuntur quadratum lateris reliqui: cuius tetra-
gonica radix eft pro ipſo lateri queſito.*

B A S I S

Ex utroque latere, per 6 bujus.

*Addet in unam ſummam quadrata laterum, compoſit quadratum Bafis: cuius radix quadrata ipſam
Bafim manifeſtat.*

Ex angulis & alterutro latere, per 7 bujus.

Vt sinus anguli dato *I* *II* *III* *IV*
lateri oppofiti } ad radius } Ita datum latus, ad bafim.

vel per 9 bujus,

Vt radius *I* *II* *III* *IV*
ad ſecantem anguli dato } Ita datum latus, ad bafim.

A N G U L I

Ex basi & latere alterutro, per 7 bujus.

Ut basis } *ad latus datum* } *Ita radius* } *ad finum anguli dato*
Ut latus } *ad basim* } *Ita radius* } *lateri oppositi.*

vel per 9 bujus,

Ut latus datum } *ad basim* } *Ita radius* } *ad secantem anguli dato*
Ut latus } *ad latus reli-* } *Ita radius* } *lateri adjacentis.*

Ex utroque crure, per 8 bujus.

Ut latus } *ad latus reli-* } *Ita radius* } *ad Tangentem anguli reliquo*
Ut latus } *quum* } *lateri oppositi.*

I N T R I A N G U L O

O B L I Q U A N G U L O

inveniuntur

A N G U L I

Ex tribus lateribus, per 13 bujus.

Ut basis Trianguli } *ad summam* } *Ita laterum dif-* } *ad basis seg-*
Ut Basis segmentum cum } *lateralium* } *ferentia* } *mentum.*
semifisse residui } *majus* } *Ita radius* } *ad secantem anguli lateri*
Ut semifisis re- } *ad latus mi-* } *Ita radius* } *majori adjacentis.*
fidis } *nus* } *ad secantem anguli lateri*
Dantur jam duo anguli: Tertius est horum duorum residuus ad semicirculum.

L A T U S E T A N G U L I D U O.

Ex duobus lateribus datis, & uno angulo obtuso non ab iis comprehenso: vel si acuto data specie alterutrius anguli ignoti, per 14 bujus.

Ut latus datum } *ad finum* } *Ita latus al-* } *ad finum ang. oppositi minoris*
dato angulo } *anguli* } *terum* } *quadrante, si species anguli*
oppositum } *datis* } *acuta sit, majoris si obtusa.*

Dantur jam duo anguli: Tertius est horum duorum residuus ad semicirculum.

Ut finus anguli al- } *ad latus oppo-* } *Ita finus an-* } *ad latus tertium.*
terutrius noti } *scium* } *guli tertii* }

ANGULUS ET DUO LATERA,

Ex duobus angulis & uno latere, per eandem.

Terius angulus est reliquus duorum datorum ad semicirculum. Itaque,

Vt sinus anguli dato latere oppositi } *ad latus datum* } *Ita sinus anguli secundi* } *ad latus op-*
teri oppositi } *ad latus datum* } *secundi* } *positum.*

Vt sinus anguli alterutrius } *ad suum latus oppositum* } *Ita sinus anguli tertii* } *ad latus tertium.*

ANGULI DUO ET LATUS

Ex duobus lateribus & angulo ab iis comprehenso, per 15 hujus.

Vt semiſſus ſumma laterum data } *ad differentiam ſumme ſemiſſus & lateris alterutrius:*

Ita Tangens ſemiſſus residui anguli ad ſemicirculum } *ad Tangentem anguli, quo angulus lateri minori oppofitus ſemifſſe dicti residui anguli ad ſemicirculum minor eſt: ſemifſſe majori major eſt.*

Vt anguli alterutrius ſinus } *ad latus op-* } *Ita ſinus anguli quaſito lateri oppofiti* } *ad latus qua-*
poſitum } *poſitum* } *ſitum.*

G E O M E T R I A E
T R I A N G U L O R U M
L I B E R IIII.

De Calculo Triangulorum Sphericorum.

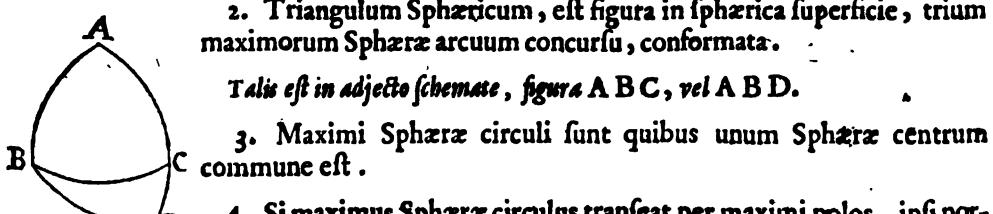
I.

ANONIS Triangulorum compotiti uſus alter eſt, in Calculo Triangulorum Sphericorum.

CSuperioris libri Theoremate primo, duplex nobis uſus Triangulorum Canonis indicatus eſt: prior in rectilineorum, posterior in Sphericorum Triangulorum Calculo. Priorie vero ratio premiſſo tractatu nobis fuſe explicata eſt: Posterioris demonstratio hoc libro continentur.

2. Triangulum Sphericum, eſt figura in sphærica superficie, trium maximorum Sphæræ arcuum concurſu, conformata.

Taliſ eſt in adjecto ſchemate, figura ABC, vel ABD.

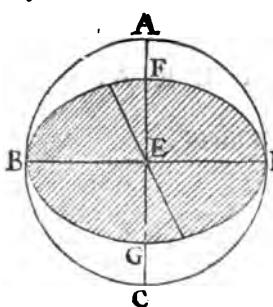


3. Maximi Sphæræ circuli ſunt quibus unum Sphæræ centrum commune eſt.

4. Si maximus Sphæræ circulus tranſeat per maximi polos, iſi normalis eſt: & contra.

Maximus circulus ABCD, tranſeat per maximi circuli BGDF polos A & D: dico circulum AB

Geometriæ Triangulorum Liber IIII.



A B C D , normalem esse circulo B G D F . Ducatur enim per centrum Sphaera E , recta B E D , ad communem intersectionem planorum B & D : secetque eam alia recta A E C normaliter per centrum E , & polos A & C ; erit hac per 4 undecimi Euclidis plano circuli A B C D normalis . Itaque per 18 ejusdem , planum circuli A B C D , i.e. ipse circulus A B C D : est normalis piano circuli B G D F , i.e. ipsi circulo B G D F ; quod erat demonstrandum . Conversa ex eadem demonstrazione perspicua est . Diameter enim B E D , secat axin A E D in E centro normaliter , per 4 undecimi Euclidis : puncta autem A & D , sunt poli circuli B G D F , ex poli definitione ; per quos necessario transfit circulus A B C D , per conversam decimaoctava undecimi elementorum . Itaque maximus Sphaera circulus A B C D , maximo B G D F normalis , transfit per polos ejus : quod erat demonstrandum .

Π Ο Ρ Ι Σ Μ Α Τ Α δυο .

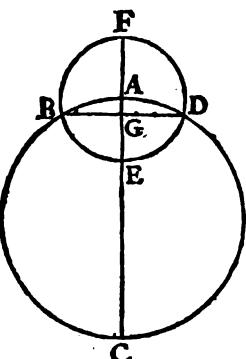
Itaque demissus à polo circuli maximi , in circumferentiam suam arcus , dictæ circumferentiaz normalis est .

Sit enim in figura superiori A B arcus maximi circulus , demissus in circumferentiam B G D F à polo ejusdem A : erit eidem normalis . Nam cum arcus A B , transeat A polum circuli B G D F , vel saltē in eo desinas , consequitur eidem normalem esse .

Punctum vero concursus duorum arcuum maximi circuli , vel unius quadrantis terminus , normaliter è circulo maximo educitorum , est ejusdem circuli polus .

Sic in eodem diagrammate , A punctum concursus duorum arcuum B A & D A , educitorum normaliter è circulo maximo B G D F : vel A , terminus quadrantis B A vel D A ex eodem circulo normaliter educiti , est ejusdem circuli polus . Nam cum B A & D A signat circulo B G D F normales sint ex thesi , necesse per polos transiant , vel in polo concurrunt : & proinde punctum concursus arcuum B A & D A , vel terminus quadrantis alterutrius , est circuli B G D F polus .

5. Si maximus Sphaeræ circulus , transeat per minoris circuli polum , eidem normalis est .

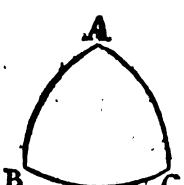


Maximus Sphaera circulus A B C D , transeat per A polum circuli minoris B E D F : dico maximum minori normalem esse . Maximus enim circuli diameter A E C , est normalis diametro minoris B G D per 3 tertii elementorum . Itaque & circulus maximus A B C D , minimo B E D F normalis per 18 undecimi Euclidis : quod erat demonstrandum .

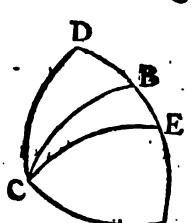
6. Triangulum Sphaericū , rectangulum est , aut obliquangulū .

7. Rectangulum est quod angulum habet rectum .

8. Anguli amplitudinem in Sphaericō Triangulo , mensurat arcus maximi circuli , ex angulo tanquam polo descriptus dictum angulum subtendens .



Ita in adjuncta Diaphraha , arcus B C , mensurat angulum B A C : est enim arcus magni circuli , ex angulo A , tanquam polo descriptus , ipsum angulū subtendens .



9. Si Trianguli rectanguli latus alterum , sit quadrans circuli , oppositus angulus rectus est ; si quadrante majus , obtusus ; si minus , acutus ; & contra .

Latus rectanguli Trianguli alterum , vocamus arcum alterutrum qui rectum angulum continent . Esto igitur Triangulum Sphericum A B C , rectangulum ad A : Sitque A B latus circuli quadrans . Dico angulum B C A oppositum , rectum esse . Nam per secundum porisma quarti hujus , B est polus circumferentiae C A : per quem transfit arcus B C . Itaque per primum porisma ejusdem , arcus B C est normalis circumferentia C A : & proinde angulus ad C rectus . Fiat vero A D latus quadrante majus , & arcus A B circuli quadrans : erit angulus A B C A rectus , per primam hujus Theorematis partem ; & proinde D C A obtusus

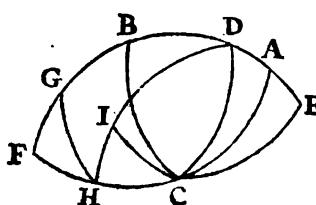
Geometriæ Triangulorum Liber IIII.

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sus (angulus enim DCA, major est angulo BCA) tandem statuatur latus AE quadrante minus, & arcus AB circuli quadrans: erit angulus BCA rectus per primam partem hujus, major angulo ECA; & proinde angulus ECA acutus est.

Conversa eadem ratione demonstratur. sint enim in eodem Triangulo, anguli BCA, & BAC recti: erunt opposita latera BA, & BC, circuli quadrantes. Arcus enim BA, & BC, egredientes normaliter ex peripheria circuli maximi CA, concurrunt in B, ejusdem polo, per secundum porisma quarti hujus: ideoque quadrantes sunt maximorum circulorum. Simili ratione demonstratur DA, latus, majus esse circuli quadrante, si angulus ad C obtusus sit; minus, si acutus. Nam si angulus DCA constituantur obtusus, erit BCA rectus, & proinde latus DA majus latere BA circuli quadrante: si ECA constituantur acutus, erit BCA rectus; & proinde EA minus BA quadrante: quod erat ostendendum.

10. Si trianguli rectanguli latus alterum sit quadrans circuli, etiam basis quadrans est: si vero utrumque latus quadrante circuli majus sit, aut minus, basis quadrante minor est; quod si latus unum circuli quadrante majus sit, reliquum minus, basis quadrante major est: & contra.



Theorematis hujus partes tres sunt. Prima, basin Trianguli rectanguli esse quadrantem circuli, si latus alterum sit circuli quadrans; & contra. Esto igitur Sphericum Triangulum ABC, rectangulum ad A: sique latus AB circuli quadrans. Dico BC basin etiam circuli quadrantem esse. Nam per primum Theorema, angulus ad C rectus est: & proinde arcus AB & CB, normaliter egrediuntur ex CA circumferentia, concurrunt autem in B pola. Itaque per 2 porisma quarti hujus, maximorum circulorum quadrantes sunt.

Conversa hujus partis perspicua est. Sit enim angulus ad A rectus, & BC circuli quadrans. Dico alterium laterum etiam circuli quadrantem esse: polo enim B, describatur maximus circulus, secturus circumferentiam BA in A; vel supra A in D; infra in E: si fecerit in A, constat B'A latus quadrantem esse per secundum porisma quarti hujus. Si vero in D, aut E punctis, anguli ad D & E recti sunt per primum porisma ejusdem: angulus autem ad A rectus est ex Thesi; quare per secundum porisma ejusdem, C est polus circumferentie BDAE, & latus CA circuli quadrans.

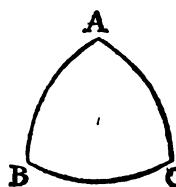
Secunda hujus Theorematis pars est: Basin quadrante minorem esse, si utrumque Trianguli rectanguli latus, quadrante majus sit, aut minus: & contra. Assumatur igitur & bic Triangulum ABC, rectangulum ad A: continenturque latera AB & AC, in F oppositum polum; componentur duo Triangula, ABC, & FBC, invicem equalia. Ducto vero arcu GH, per puncta G & H: fiet GH basis, communis Triangulo GAH rectangulo, habenti latera AG & AH, quadrante circuli AB, vel AC, majora; Itemque Triangulo GFH rectangulo reliquo habenti latera FG & FH quadrante circuli FB vel FC minora; basis vero GH erit minor BC quadrante circuli: rectos angulos ad F, & A, per 8 hujus mensurante. Secus enim si non sit, vel major erit BC arcu, vel aquilis ipsi. Sed major esse nequit: quia Triangulum ABC, ad omnes angulos rectangulum, non potest capere latus recto majus. Aequalis esse nequit, quia neutrius Trianguli latus circuli quadrans est: consequitur igitur basin GH, quadrante minorem esse.

Conversa hujus partis etiam facilis est. Sit enim basis quadrante minor: dico utrumque latus Trianguli rectanguli dati, quadrante majus, aut minus esse. Nam si non sit; unum quadrans est; vel unum quadrante majus, & reliquum minus. Atqui si unum latus quadrans sit: est & basis quadrans. Vel si unum latus quadrante majus sit, reliquum minus: basis quadrante major est. Utrumque est contra Thesis. Ergo utrumque latus, vel quadrante majus, vel minus est. Prioris ratio ex prima hujus Theorematis parte clara est: posterioris ex tercia: qua docet,

Basin quadrante circuli majorem esse, si unum rectanguli Trianguli latus sit quadrante circuli majus, reliquum minus: & contra. Assumatur enim & bic Triangulum DAH, rectangulum ad A: cuius latus AD, sit minus AB circuli quadrante; & reliquum AH, majus AC circuli quadrante. Dico DH basin, etiam quadrante circuli majorem esse: & contra. Arcus enim AC, est circuli quadrans ex fabrica: quemadmodum & DC per secundum porisma quarti hujus. Quare si polo D, in C describatur arcus maximi circuli CI; secabit DH basin in I, proinde DI quadrans erit, per citatum porisma, & DH quadrante major.

Conversa hujus partis similiter patet; latus alterum Trianguli rectanguli quadrante majus esse, reliquum minus, si basis quadrante major sit. Secus enim si non sit: erunt latera vel circuli quadrantes; & tunc basis est quadrans, per primam hujus Theorematis partem: vel utrumque latus erit majus quadrante, vel minus, & tum basis quadrante minor est, per secundam hujus Theorematis partem. Sed utrumque est contra Thesin. Itaque latus unum quadrante majus, reliquum minus est: que fuere demonstranda.

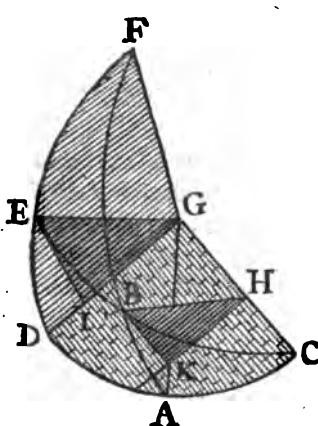
11. Si Trianguli rectanguli alteruter angulorum in basi rectus sit; basis est circuli quadrans; si uterque vel acutus vel obtusus sit, basis est quadrante minor: si vero alter eorum acutus sit, & reliquo obtusus, basis quadrante major est: & contra.



Sit Triangulum A B C rectangulum ad C. Dico A B basis, circuli quadrante esse, si alteruter angulorum in basi, A, aut B rectus sit: quadrante minorem si uterque vel acutus, vel obtusus sit; majorem, si alter acutus, reliquo obtusus sit: & contra. Si enim alteruter angulorum A, vel B rectus sit: alterutrum laterum circuli quadrans est per 9 hujus; ergo per 10 ejusdem, basis A B etiam circuli quadrans est. Si uterque angulus A & B similiter acutus sit, aut obtusus: utrumque latus A C, & C B, per nonam hujus, quadrante majus, vel minus est; ergo per 10 ejusdem, basis A B quadrante minor est. Quod si alteruter angulorum A & B acutus sit, reliquo obtusus: per 9 hujus, alterutrum laterum quadrante circuli minus, reliquum majus est; ergo per 10 ejusdem basis A B quadrante major est.

Conversa similiter probatur. sit enim basis A B circuli quadrans, alteruter angulorum A aut B rectus est: Nam per decimam hujus latus alterutrum quadrans circuli est, ergo per 9 ejusdem angulus alter rectus est. Si vero A B basis quadrante minor sit: uterque angulus A & B vel acutus, vel obtusus est; nam per 10 hujus utrumque latus vel majus est vel minus quadrante. Ergo per 9 ejusdem, uterque angulus vel acutus vel obtusus est. Demum si A B basis quadrante major sit, alteruter angulorum A, aut B acutus est, reliquo obtusus. Nam per 10 hujus, latus unum quadrante minus, reliquum majus est: ergo per 9 ejusdem, angulus alter acutus, reliquo obtusus est; qua fuerunt ostendenda.

12. Si quadrans maximi circuli, ad quadrantem maximi inclinatus fuerit, & ab inclinato perpendiculares duo descendant, quorum alter utriusque quadrantis terminum secet: sinus recti segmentorum quadrantis inclinati, ab inclinationis angulari punto, perpendicularium rectis sinibus proportionales sunt.



Esto C B E maximi circuli quadrans, inclinatus ad C A D maximi circuli quadrantem; & ab C B E inclinato, descendant duo arcus E D & B A: quorum alter E D, secet E & D terminum utriusque quadrantis C B E & C A D. Dico rectas B H & E G, sinus rectos segmentorum C B, & C E, proportionales esse rectis B K & E I, rectis sinibus perpendicularium arcum B A & E D. Triangula enim G I E & H K B, sunt equiangula ob rectos angulos ad I, & K, per 7 primi hujus; & similem ad G & H, inclinationis scilicet superficiei quadrantis G E C, ad superficiem quadrantis G D C ang. Itaque per quartum sexti elementorum, latera qua subter aequalis eos angulos sunt, B H & E G: Item B K & E I, sunt proportionalia; quad era demonstrandum.

Π Ο Ρ Ι Σ Μ Α Τ Α οθο.

Primo itaque, in rectangulo Triangulo, unicui rectum habente, ex data basi, & angulo alterutro obliquo, invenitur latus oppositum. Radius enim est ad sinum basis: ut sinus anguli ad sinum lateris oppositi. Vel, Radius est ad secantem complementi basis; ut secans complementi anguli ad secantem complementi lateris oppositi. Vel sinus basis est ad radium; ut secans complementi anguli, ad secantem complementi lateris oppositi. Vel secans complementi basis est ad radium: ut sinus anguli ad sinum lateris oppositi.

Esto in praecedenti Diagrapha, A B C Triangulum rectangulum, unicum rectum habens ad A per primum porisma quarti hujus: deturque B C basis ejus, partium 60; & angulus A C B part. 30. Invenietur A B latus oppositum partium 25 39' 32". Nam per quartam sexti elementorum, & 19 septimi, Vt E G

$$\left. \begin{array}{l} \text{Vt EG radius} \\ 10000000 \end{array} \right\} \left. \begin{array}{l} \text{ad BH sinum} \\ \text{basis BC} \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{Ita EI sinus arcus} \\ \text{ED vel anguli} \\ \text{ACB per 8 bujus} \\ 5000000, ad \end{array} \right\} \left. \begin{array}{l} \text{BK sinum lateris oppositi} \\ \text{BA 4330127 par-} \\ \text{tium 2539'32''} \end{array} \right\}$$

Ergo latus AB est partium 2539'32": quadrante minus per 9 bujus, quia angulus oppositus ACB acutus est.

Demonstratum vero est 20 Theoremate primi bujus, sinum rectum peripheria ad radium esse: ut radius ad secantem complementi. Itemque, secantes peripheriarum, complementorum suorum rectis sinibus proportionales esse. Itaque per secundum porisma Theorematis citati,

$$\left. \begin{array}{l} \text{Vt radius} \\ 10000000 \end{array} \right\} \left. \begin{array}{l} \text{ad secantem com-} \\ \text{pletum. basis BC} \\ 11547004 \end{array} \right\} \left. \begin{array}{l} \text{Ita secans compl.} \\ \text{anguli dati} \\ 20000000 ad \end{array} \right\} \left. \begin{array}{l} \text{secantem compl. la-} \\ \text{teris oppositi AB} \\ 23094008 par. 64 \\ 20'28'' \end{array} \right\}$$

Quare latus AB est partium 253932 ut supra.

Aliter per primum porisma Theorematis citati,

$$\left. \begin{array}{l} \text{Vt sinus basis} \\ 8660254, ad \end{array} \right\} \left. \begin{array}{l} \text{Radium 10000000} \end{array} \right\} \left. \begin{array}{l} \text{Ita secans com-} \\ \text{pletum. ang. dati} \\ 20000000, ad \end{array} \right\} \left. \begin{array}{l} \text{secantem compl. la-} \\ \text{teris oppositi AB} \\ 23094008. \end{array} \right\}$$

Aliter per secundum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt secans comp. basis} \\ 11547004 ad \end{array} \right\} \left. \begin{array}{l} \text{Radium 10000000} \end{array} \right\} \left. \begin{array}{l} \text{Ita sinus anguli dati} \\ 5000000, ad \end{array} \right\} \left. \begin{array}{l} \text{Sinum lateris oppositi} \\ AB 4330127. \end{array} \right\}$$

Secundo, data basi, & latere alterutro, exquiritur angulus oppositus. Nam ut sinus basis est ad radium; ita sinus lateris dati ad sinum anguli oppositi. Vel, ut secans complementi basis est ad radium; ita secans complementi lateris, ad secantem complementi anguli oppositi. Aut, ut radius est ad sinum basis; ita secans complementi lateris, ad secantem complementi anguli oppositi. Aut, ut radius est ad secantem complementi basis: ita sinus lateris, ad sinum anguli oppositi.

Reento superiori Triangulo ABC, sit basis BC partium 60: & latus AB partium 2539'32. Invenietur angulus A C B oppositus, partium 30. Nam per quartam sexti & 19 septimi Euclidis.

$$\left. \begin{array}{l} \text{Vt BH sinus basis} \\ BC 8660254 \end{array} \right\} \left. \begin{array}{l} \text{ad EG radium} \\ 1000000, ita \end{array} \right\} \left. \begin{array}{l} \text{BK sinum lateris AB} \\ 4330127, est ad \end{array} \right\} \left. \begin{array}{l} \text{EI sinus arcus ED} \\ \text{vel anguli ACB} \\ 5000000. \end{array} \right\}$$

Angulus itaque A C B questus est partium 30: acutus per 9 bujus; quia latus oppositum est minus circuli quadrante.

Aliter per secundum porisma 20 primi bujus,

$$\left. \begin{array}{l} \text{Vt secans compl. basis} \\ 11547004 ad \end{array} \right\} \left. \begin{array}{l} \text{Radium 10000000, ita} \end{array} \right\} \left. \begin{array}{l} \text{Secans compl.} \\ \text{lateris dati} \\ 23094008 ad \end{array} \right\} \left. \begin{array}{l} \text{Secantem compl. ang. op-} \\ \text{positi 20000000,} \\ \text{partium 60.} \end{array} \right\}$$

Itaque ipse angulus est partium 30.

Aliter per primum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt radius 10000000} \end{array} \right\} \left. \begin{array}{l} \text{ad sinus basis} \\ 8660254, ita \end{array} \right\} \left. \begin{array}{l} \text{Secans complem.} \\ \text{lateris dati} \\ 23094008, ad \end{array} \right\} \left. \begin{array}{l} \text{Secantem complem.} \\ \text{anguli oppositi} \\ 20000000. \end{array} \right\}$$

Alio per secundum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad secantem complem.} \\ \text{basis } 11547004, \text{ ita} \end{array} \right\} \left. \begin{array}{l} \text{Sinus lateris dati} \\ 4330127, \text{ ad} \end{array} \right\} \left. \begin{array}{l} \text{Sinum anguli oppo-} \\ \text{siti } 5000000 \\ \text{partium } 30. \end{array} \right\}$$

Tertio, dato latere & angulo huic opposito, investigatur basis, si constiterit quadrante major sit an minor. Nam ut sinus anguli est ad radium; ita sinus lateris, ad sinum basis. Aut, ut secans complementi anguli est ad radium; ita secans complementi lateris, ad secantem complementi basis. Vel, ut radius est ad sinum anguli; ita secans complementi lateris est, ad secantem complementi basis. Aut, ut radius est ad secantem complementi anguli: ita sinus lateris ad sinum basis.

Assumpto & hic superiori Triangulo, datur latus A B partium 25 39' 32": & angulus B C A oppositus partium 30; erit B C basis part. 60. si fuerit quadrante minor: vel 120 si major. Nam per 4 sexti, & 19 sept. Euclidis,

$$\left. \begin{array}{l} \text{Vt E I sinus angul. E C D} \\ 5000000 \text{ ad} \end{array} \right\} \left. \begin{array}{l} \text{EG radium} \\ 10000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita B K sinus la-} \\ \text{teris A B dati} \\ 4330127 \end{array} \right\} \left. \begin{array}{l} \text{ad B H sinum basis B C} \\ 8660254 \text{ minorem} \\ \text{quadrante partium 60.} \end{array} \right\}$$

Aut per secundum porisma 20 primi hujus,

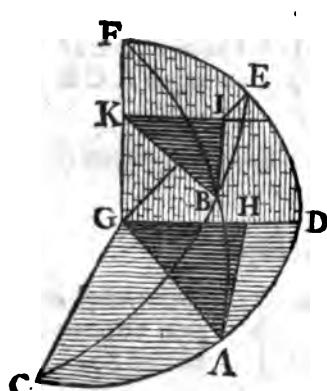
$$\left. \begin{array}{l} \text{Vt secans compl.} \\ \text{ang. } 20000000 \end{array} \right\} \left. \begin{array}{l} \text{ad radium } 10000000 \\ 5000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita secans compl. lateris dati} \\ 23094008, \text{ ad} \end{array} \right\} \left. \begin{array}{l} \text{Secantem complemen. basis} \\ 11547004 \text{ part. } 30. \text{ ergo} \\ \text{basis est partium } 60. \end{array} \right\}$$

Vel per primum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad } 5000000 \end{array} \right\} \left. \begin{array}{l} \text{Sinum ang. dati} \\ 5000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita secans compl. lateris dati} \\ 23094008, \text{ ad} \end{array} \right\} \left. \begin{array}{l} \text{Secantem complem. basis} \\ 11547004. \end{array} \right\}$$

Vel per secundum porisma citate,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad } 20000000 \end{array} \right\} \left. \begin{array}{l} \text{Secantem complem. ang.} \\ 11547004, \text{ ita} \end{array} \right\} \left. \begin{array}{l} \text{Sinus lateris dati} \\ 433012 \end{array} \right\} \left. \begin{array}{l} \text{ad sinum basis } 8660254 \\ \text{ut supra.} \end{array} \right\}$$



Quarto, dato latere alterutro & basi, innotescit latus reliquum: sinus enim complementi lateris dati est ad radium; ut sinus complementi basis, ad sinum complementi lateris reliqui. Vel, secans lateris dati est ad radium; ut secans basis ad secantem reliqui lateris. Vel, radius est ad sinum complementi lateris; ut secans basis ad secantem lateris alterius. Vel, radius est ad secantem lateris; ut sinus complementi basis, ad sinum complementi lateris reliqui.

Sit & hic Trianguli A B C, latus A B partium 25 39' 32": & BC basis partium 60. Invenietur reliquum latus AC, partium 56, 18', 35". Nam per quartam sexti, & 19 septimi Euclidis,

$$\left. \begin{array}{l} \text{Vt B K sinus arcus FB,} \\ \text{comple. lateris A B dati} \\ 9013880 \end{array} \right\} \left. \begin{array}{l} \text{ad A G radium} \\ 10000000, \text{ ita} \end{array} \right\} \left. \begin{array}{l} \text{BI sinus arcus BE,} \\ \text{i. compl. basis BC} \\ 5000000, \end{array} \right\} \left. \begin{array}{l} \text{ad A H sinus arcus AD} \\ \text{com. lat. AC } 5547002 \\ \text{partium } 334125. \end{array} \right\}$$

Itaque A C latus est partium 56 18' 35", quadrante minus per 10 bujus, quia basis cum reliquo latere signatim quadrante minor est.

Ali-

Aliter per 2. porisma vice simile primi hujus,

*Vt secans lateris } ad radium { Ita secans basis } ad secantem reliqui lateris 18027760
dati 11094005 } 10000000 { 20000000 } partium 56 18' 35".*

Vel per primum porisma ejusdem,

*Vt radius } ad sinum complem. lateris { Ita secans basis 20000000 } ad secantem reliqui
10000000 } dati 9013880 { 5000000 } lateris 18027760.*

Vel per secundum porisma ejusdem,

*Vt radius } ad secantem lateris dati { Ita sinus complem. basis } ad sinum complem. lateris
10000000 } 11094005 { 5000000 } quefit 5547002.*

Quinto, dato utroque latere investigatur basis. Nam, ut radius est ad sinum complementi lateris alterutrius: ita sinus complementi lateris reliqui, est ad sinum complementi basis. Vel, radius est ad secantem lateris alterutrius, ut secans lateris reliqui ad secantem basis. Vel, sinus complementi lateris alterutrius est ad radium; ut secans lateris reliqui, ad secantem basis. Vel secans lateris alterutrius est ad radium; ut sinus complementi lateris reliqui, ad sinum complementi basis.

Detur in Triangulo A B C, latus A B partium 25 39' 32": & A C reliquum latus partium 56 18' 35". invenietur basis B C partium 60. Nam per 4 sexti, & 19 septimi Euclidis,

*Vt AG radius } ad B K sinum arcus FB { Ita AH sinus arcus DA } ad BI sinum arcus EB
10000000 } .i. compl. lateris A B { .i. compl. lateris D C } .i. compl. basi B C
9013880 } 5547002 } 5000000 part. 30.*

Ergo basis B C est partium 60, minor circuli quadrante per 10 hujus, quia utrumque latus sigillatum quadrante minus est.

Aliter per secundum porisma 20 primi hujus,

*Vt radius } ad secantem lateris A B { Ita secans lateris A C } ad secantem basis 20000000,
10000000 } 11094005 { 18027760 } part. 60.*

Vel per primum porisma ejusdem,

*Vt sinus complem. lat. A B } ad radium { Ita secans reliqui lateris } ad secantem basis
9013880 } 10000000 { 18027760 } 20000000.*

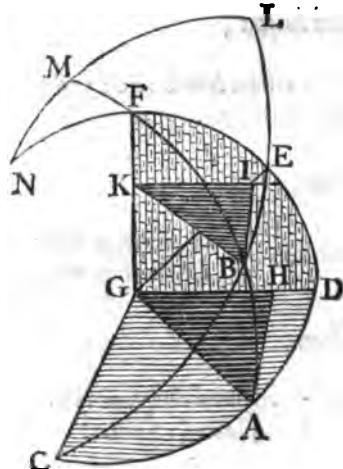
Vel per secundum porisma ejusdem,

*Vt secans lat. A B } ad radium { Ita sinus complem. lateris } ad sinum complem. basis
11094005 } 10000000 { reliqui 5547002 } 5000000.*

Sexto, dato latere & angulo adjacente, innoteat obliquus alter. Radius enim est ad sinum complementi lateris: ut sinus anguli, ad sinum complementi reliqui. Aut, radius est ad secantem lateris dati; ut secans complementi anguli, ad secantem anguli reliqui. Aut, sinus complementi lateris dati est ad radium; ut secans complementi anguli, ad secantem reliqui. Aut, secans lateris dati est ad radium; ut sinus anguli, ad sinum complementi reliqui.

Repetita postrema Trianguli nostri figura, detur latus A C partium 56 18' 35": angulusque ei adjacentes A C B partium 30. Invenietur reliquis obliquus A B C, part. 73 53' 52" & paucus plus.

Geometriæ Triangulorum Liber IIII.



Continuerunt enim arcus, BE in L; BF in M: & EF in N; ut BL, BM, & EN, quadrantes sint maximorum circulorum. Facto vero N polo, describatur maximi circuli quadrans NML, perterminos quadrantum BM, & BL. Manifestum est angulum ad M, in Triangulo NMF rectum esse, per primum porisma quarti hujus: & basis FN, complementum esse arcus FE; & proinde aqualem arcus ED. Item angulum ad F, aqualem esse angulo AFD, vel arcui AD, complemento scilicet lateris AC. Quare cum in eodem Triangulo NMF rectangulo, detur basis FN, equalis angulo ACB: & angulus ad F equalis complemento lateris AC; dabitur etiam oppositum angulo latus NM, complementum scilicet arcus ML, angulum ad B quecumque subtendens. Nam per primum porisma hujus,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad basim FN i.} \\ \text{sinu ang. ACB} \\ 5000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita ang. MFN i.} \\ \text{sinu comp. lat. AC} \\ 5547002 \end{array} \right\} \left. \begin{array}{l} \text{ad MN i. sinu comp. an-} \\ \text{guli ad B, } 2773501 \text{ par-} \\ \text{tium } 16^{\circ} 6' 8'' \text{ fere.} \end{array} \right\}$$

Ergo angul. ABC est partium $73^{\circ} 53' 52''$; ratus per 9 hujus, quia oppositum ei latus AC quadrante minus est.

Alio per 2 porisma 20 primi hujus,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad secantem com-} \\ \text{pl. ang. dati} \\ 20000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita secans lateris dati} \\ 18027760 \end{array} \right\} \left. \begin{array}{l} \text{ad secantem angul. reliqui} \\ 36055520 \text{ part. } 73^{\circ} \\ 52'', \text{ ut supra.} \end{array} \right\}$$

Vel per primum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt sinus anguli dati} \\ 5000000 \end{array} \right\} \left. \begin{array}{l} \text{ad radium } 10000000 \\ \text{ad secantem lateris dati} \\ 18027760 \end{array} \right\} \left. \begin{array}{l} \text{ad secantem anguli re-} \\ \text{liqui } 36055520. \end{array} \right\}$$

Vel per secundum porisma ejusdem,

$$\left. \begin{array}{l} \text{Vt secans comp. ang. dati} \\ 20000000 \end{array} \right\} \left. \begin{array}{l} \text{ad radium } 10000000 \\ \text{ad secantem lateris dati} \\ 5547002 \end{array} \right\} \left. \begin{array}{l} \text{Ita sinus compl.} \\ \text{lateris dati} \\ \text{anguli reliqui} \end{array} \right\} \left. \begin{array}{l} \text{ad sinus compl.} \\ \text{anguli reliqui} \\ 2773501 \end{array} \right\}$$

Septimo, dato latere, & angulo opposito; datur obliquus reliqui, si species ejus nota sit. Sinus enim complementi lateris dati est ad radium; ut sinus complementi anguli dati ad sinum reliqui. Vel, secans lateris dati est ad radium; ut secans anguli dati, ad secantem complementi reliqui. Vel, radius est ad sinum complementi lateris dati: ut secans anguli dati, ad secantem complementi reliqui. Vel, radius est ad secantem lateris; ut sinus complementi anguli dati, ad sinum reliqui.

Detur in Triangulo ABC latus AB partium $25^{\circ} 39' 32''$: & angulus ei oppositus ACB partium 30 ; cum specie reliqui ad B acuta. Invenietur ipse angulus ad B partium $73^{\circ} 53' 52''$. Nam in Triangulo FMN rectangulo, datur latus FM, aquale lateri AB: & basis NF equalis arcus DE, .i. angulo ACB. Ergo & reliquum latus NM, .i. complementum arcus ML, vel anguli ad B, per quartum porisma hujus immotescit. Nam,

$$\left. \begin{array}{l} \text{Vt sinus com. MF} \\ .i. AB \text{ lat. dati} \\ 9013880 \end{array} \right\} \left. \begin{array}{l} \text{ad radium } 10000000 \\ \text{ad sinus comp. basis} \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{Ita sinus comp. basis} \\ NF .i. ang. dati \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{ad sinus ML .i. ang. ad} \\ B 9607690 \text{ part. } 73 \\ 53' 52'', \text{ acuti ex ibi.} \end{array} \right\}$$

Vel

Vel per secundum porisma 20 primi bujus,

$$\begin{array}{l} \text{Vi secans A B} \\ \text{lateris dati} \\ 11094005 \end{array} \left\{ \begin{array}{l} \text{ad radium 10000000} \\ \text{Ita secans ang.} \\ \text{A C B dati} \\ 11547004 \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem complementi an-} \\ \text{guli reliqui 10408330 par-} \\ \text{tium 16' 8''}. \end{array} \right\}$$

Ergo ipse angulus est partium 73 53' 52".

Vel per primum porisma ejusdem,

$$\begin{array}{l} \text{Vi radius 10000000} \\ \text{ad finum compl. lateris} \\ \text{dati 9013880} \end{array} \left\{ \begin{array}{l} \text{Ita secans anguli} \\ \text{ACB 11547004} \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem compl. ang.} \\ \text{reliqui 10408330}. \end{array} \right\}$$

Vel per secundum porisma 20 primi bujus,

$$\begin{array}{l} \text{Vi radius 10000000} \\ \text{ad secantem lateris A B} \\ 11094005 \end{array} \left\{ \begin{array}{l} \text{Ita finus complem.} \\ \text{ang. A C B dati} \\ 8660254 \end{array} \right\} \left\{ \begin{array}{l} \text{ad finum anguli reli-} \\ \text{qui 9607690} \end{array} \right\}$$

Postremo, dato utroque angulo obliquo, datur etiam latus alterutrum. Sinus enim anguli unius, se habet ad radium; ut sinus complementi reliqui, ad finum complementi lateris oppositi. Aut, secans complementi anguli unius est ad radium; ut secans alterius, ad secantem lateris oppositi. Vel, radius est ad finum anguli alterutrius; ut secans anguli reliqui, ad secantem lateris oppositi. Vel, radius est ad secantem complementi anguli unius; ut finus complementi alterius, ad finum complementi lateris oppositi.

Manente postremo diagrammate, detur in Triangulo A.B.C rectangulo, uterque obliquus angulus ad B & C: dabitur etiam alterutrum latus. Nam in Triangulo M.F.N rectangulo, datur latus M.N, complementum arcus L.M, subtendens angulum ad B: & basis N.F, complementum scilicet arcus F.E, .i. arcus E.D, subtendens angulum ad C. Ergo & angulus ad F oppositus, .i. arcus D.A, vel complementum lateris A.C invenietur. Nam per 2 porisma bujus,

$$\begin{array}{l} \text{Vi finus basis F N} \\ \text{.i. ang. ad C,} \\ 5000000 \end{array} \left\{ \begin{array}{l} \text{ad radium} \\ 10000000 \end{array} \right\} \left\{ \begin{array}{l} \text{Ita finus lateris M N} \\ \text{.i. complem. ang. ad} \\ B 2773501 \end{array} \right\} \left\{ \begin{array}{l} \text{ad M F N finum ang. oppositi} \\ \text{.i. complem. lateris A C} \\ 5547002 partium 16' 8'', fere. \end{array} \right\}$$

Ergo ipsam latus A.C est partium 73 53' 52" paulo plus: quadrante minus per 9 bujus, quia angulus oppositus acutus est.

Alicet per 2 porisma 20 primi bujus,

$$\begin{array}{l} \text{Vi secans compl.} \\ \text{anguli ad C} \\ 20000000 \end{array} \left\{ \begin{array}{l} \text{ad radium 10000000} \\ \text{Ita secans ang.} \\ \text{reliqui ad B} \\ 36055520, \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem lateris oppositi,} \\ 18027760, par. 73 53' 52'' \\ \text{paulo plus, ut supra.} \end{array} \right\}$$

Vel per primum porisma 20 primi bujus,

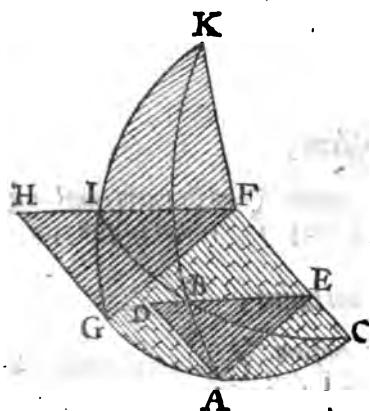
$$\begin{array}{l} \text{Vi radius 10000000} \\ \text{ad finum ang. ad} \\ C 5000000 \end{array} \left\{ \begin{array}{l} \text{Ita secans ang. ad B} \\ 36055520 \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem lateris oppositi} \\ 18027760. \end{array} \right\}$$

Vel per secundum porisma ejusdem,

$$\begin{array}{l} \text{Vi radius 10000000} \\ \text{ad secantem compl. ang.} \\ \text{ad C 20000000} \end{array} \left\{ \begin{array}{l} \text{Ita finus compl.} \\ \text{anguli reliqui} \\ 2773501 \end{array} \right\} \left\{ \begin{array}{l} \text{ad finum compl.} \\ \text{lateris oppositi} \\ 5547002. \end{array} \right\}$$

Geometriae Triangulorum Liber IIII.

13. Si quadrans maximi circuli, quadrantem maximi secet, & à secante, duo arcus perpendicularares secto ducantur; quorum alter per utriusque quadrantis terminum transeat; sinus recti segmentorum quadrantis secti, à punto sectionis, perpendicularium tangentibus proportionales sunt.



Proposita.

Maximi circuli quadrans IBC, secet GAC quadrantem maximi in C: & ab IBC secante, descendant perpendicularares arcus duo, IG & BA; quorum alter IG, transeat per terminum utriusque quadrantis I & G. Dico sinus rectos GF & AE, segmentorum CI & CA: proportionales esse tangentibus HG & DA, perpendicularium I G & BA. Triangula enim HGF, & DAE, sunt equiangula: ob rectos angulos ad G & A, per 15 primi hujus; Communem ad F & E, inclinationis scilicet angulum superficies quadrantis secantis, ad superficiem quadrantis secti. Itaque per quartam sexti elementorum sunt lateram proportionalium. Quare ut GF, ad AE: HG ad DA, quod erat demonstrandum.

Primo, igitur in rectangulo Triangulo, dato latere & angulo adjacenti, investigatur latus reliquum. Radius enim est ad sinum lateris dati; ut tangens anguli adjacentis, ad tangentem reliqui lateris. Vel, radius est ad secantem complementi lateris: ut tangens complementi anguli adjacentis, ad tangentem complementi lateris alterius. Vel, sinus lateris dati est ad radium: ut tangens complementi anguli adjacentis, ad tangentem complementi lateris reliqui. Vel secans complementi lateris dati, est ad radium: ut tangens anguli adjacentis, ad tangentem reliqui lateris.

Assumpto & bic Triangulo ABC rectangulo, datum latus AC partium 56 18' 35": & angulus adjacens ad C partium 30. invenietur reliquum latus AB, part. 25 39' 32". Nam per 4 sextas & 19 septim. Euclidis,

$$\left. \begin{array}{l} \text{Vt } GF \text{ radius} \\ 10000000 \end{array} \right\} \text{ad AE sinum} \quad \left. \begin{array}{l} \text{lateris AC} \\ 8320482 \end{array} \right\} \quad \left. \begin{array}{l} \text{Ita GH tan-} \\ \text{gents arcus IG} \\ \text{i.e. ang. ad C;} \\ 5773502 \end{array} \right\} \quad \left. \begin{array}{l} \text{ad AD tangentem later. AB} \\ 4803831 \\ \text{part. 25 39' 32". quadr. mino-} \\ \text{ris per 9 hujus quia ang. oppos. acu-} \\ \text{bus est.} \end{array} \right\}$$

Demonstratum vero est 20 Theoremate prius hujus, secantes arcum, complementorum suorum rectis sinibus: itemque 17 ejusdem, tangentes arcum complementorum suorum tangentibus proportionales esse. Itaque si loco sinuum, tangentiumque peripheriarum datarum, assumantur complementorum secantes & tangentes, manebit eadem proportio. Quare,

$$\left. \begin{array}{l} \text{Vt radius 10000000} \\ \text{12018535} \end{array} \right\} \text{ad secantē comp.} \quad \left. \begin{array}{l} \text{Ita tangens comp.} \\ \text{anguli ad C} \\ 17320508 \end{array} \right\} \text{ad tang. compl. lateris oppositi} \quad \left. \begin{array}{l} \text{AB 20816713 partium} \\ 64 20' 28". \end{array} \right\}$$

Ergo ipsum latus est partium 25 39' 32".

Vel, quia radius media proporcione est ad tangentes peripherie & complementi, per 17 prius hujus,

$$\left. \begin{array}{l} \text{Sinus lateris AC} \\ 8320482 \end{array} \right\} \text{est ad radium 10000000} \quad \left. \begin{array}{l} \text{ut tangens comp.} \\ \text{anguli dati} \\ 17320508 \end{array} \right\} \text{ad tangent. complementi} \quad \left. \begin{array}{l} \text{lateris AB oppositi} \\ 20816713 \end{array} \right\}$$

Vel,

$$\left. \begin{array}{l} \text{Secans complem. Lateris} \\ AC 12018535 \end{array} \right\} \text{est ad radium} \quad \left. \begin{array}{l} \text{ut tangens anguli ad C} \\ 5773502 \end{array} \right\} \text{ad tangent. lateris AB} \quad \left. \begin{array}{l} \text{oppositi 4803831.} \\ \text{Se-} \end{array} \right\}$$

Secundo, dato latere & angulo opposito, exquiritur reliquum latus, si constiterit quadrantene majus sit an minus. Nam ut tangens anguli dati est ad radium: ita tangens lateris oppositi, ad sinum anguli reliqui. Vel, ut tangens complemeati anguli noti, ad radium est: ita tangens complementi lateris oppositi, ad secantem complementi lateris alterius. Vel, ut radius ad tangentem anguli dati; ita tangens complementi alterius lateris, ad secantem complementi lateris oppositi. Vel, ut radius ad tangentem complementi anguli dati: ita tangens lateris oppositi, ad sinum lateris reliqui.

Manente figura superioris Trianguli, sit latus A B part. 25 39' 32": & angulus ad C oppositus partium 30; dabitur reliquum latus A C partium 56 18' 35". Nam per 4 sexii & 19 septimi Euclidis,

$$\begin{array}{l} \text{Vt GH tangen. arcus IG. i. ang. ad C } 17320508 \\ \quad \left\{ \begin{array}{l} \text{ad GF radium 10000000} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita AD tang. lateris A B oppositi 4803831} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad AE sinum lateris reliqui AC 8320482. part. 56} \\ \quad 18' 35'' \text{ si minus quadrante sit, partium vero 123 41'} \\ \quad 25'', \text{ si majus sit.} \end{array} \right\}$$

Vel per 17 & 20 primi bujus,

$$\begin{array}{l} \text{Vt tangens comp. anguli ad C } 17320508 \\ \quad \left\{ \begin{array}{l} \text{ad radium 10000000} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita tangens compl. later. oppositi AB 20816713} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad secantem compl. later. AC 12018535. par. 33 41 25.} \\ \quad \end{array} \right\}$$

Ergo si ipsum latus quadrante minus est, partium est 56 18' 35".

Vel per 17 primi bujus,

$$\begin{array}{l} \text{Vt radius 10000000} \\ \quad \left\{ \begin{array}{l} \text{ad tangen. ang. ad C 17320508} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita tang. comp. late. oppositi AB 20816713} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad secan. compl. lateris AC 12018535.} \\ \quad \end{array} \right\}$$

Vel,

$$\begin{array}{l} \text{Vt radius 10000000} \\ \quad \left\{ \begin{array}{l} \text{ad tangentem compl. anguli ad C 17320508} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita tangens later. oppositi AB 4803831} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad finum lateris reliqui AC 8320482.} \\ \quad \end{array} \right\}$$

Tertio, dato utroque latere, datur angulorum obliquorum alterutrius, sinus enim lateris alterutrius est ad radium; ut tangens reliqui lateris, ad tangentem anguli oppositi. Aut, secans complementi lateris alterutrius ad radium est: ut tangens complementi alterius lateris, ad tangentem complementi anguli oppositi. Aut, radius est ad sinum lateris alterutrius: ut tangens complementi reliqui lateris, ad tangentem complementi anguli oppositi. Vel, radius est ad secantem complementi lateris unius: ut tangens alterius, ad tangentem anguli oppositi.

Retento superiori Triangulo A B C, detur latus A B part. 25 39' 32": A C partium 56 18' 35". innenietur angulus ad C partium 30. Nam per 4 sexii & 19 septimi Euclidis,

$$\begin{array}{l} \text{Vt AE sinus Lateris AC } 8320482 \\ \quad \left\{ \begin{array}{l} \text{ad GF radium 10000000} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita AD tangens reliqui lateris A B 4803831} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad GH tangentem arc. IG. i. ang. ad C oppositi 5773502. partii 30: acui per 9 bujus, quia latus oppositum est quadrante minus.} \\ \quad \end{array} \right\}$$

Alio per 17 & 20 primi bujus,

$$\begin{array}{l} \text{Vt secans compl. lat. AC 12018535} \\ \quad \left\{ \begin{array}{l} \text{ad radium 10000000} \\ \quad \end{array} \right\} \end{array} \quad \left\{ \begin{array}{l} \text{Ita tangens comp. reliqui lateris AB 20816713} \\ \quad \end{array} \right\} \quad \left\{ \begin{array}{l} \text{ad tangentem comp. anguli ad C oppositi 17320508 par. 60.} \\ \quad \end{array} \right\}$$

Ergo ipse angulus est partium 30, ut supra.

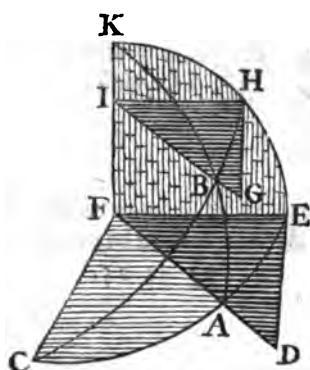
Geometriæ Triangulorum Liber IIII.

Vel per 17 primi hujus,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad sinum lat. teris AC} \\ 8320482 \end{array} \right\} \left. \begin{array}{l} \text{Ita tangens complem. reliqui lateris AB} \\ 20816713 \end{array} \right\} \left. \begin{array}{l} \text{ad tangentem comp. ang. ad C oppositi} \\ 17320508. \end{array} \right\}$$

Vel per eidem Theorema,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad secantem com. lateris AC} \\ 12018535 \end{array} \right\} \left. \begin{array}{l} \text{Ita tang. reliqui lat. AB} \\ 4803831 \end{array} \right\} \left. \begin{array}{l} \text{ad tangentem ang. ad C oppositi} \\ 5773502. \end{array} \right\}$$



Quarto, data basi & angulo, investigatur latus adjacens. Nam ut sinus complementi anguli dati ad radium : ita tangens complementi basis est, ad tangentem complementi lateris dato angulo adjacentis. Vel ut secans anguli dati est ad radium : ita tangens basis, ad tangentem lateris dato angulo adjacentis. Aut, radius est ad sinum complementi anguli dati ; ut tangens basis ad tangentem lateris angulo dato adjacentis. Aut, radius est ad secantem anguli dati : ut tangens complementi basis, ad tangentem complementi lateris dato angulo adjacentis.

Assumpto & hic Triangulo ABC rectangulo, detur basis BC part. 60 : & angulus ad C partium 30. invenietur latus AC part. 56 18' 35". Nam per quartam sexti & 19 septimi Euclidis.

$$\left. \begin{array}{l} \text{Vt IH sinus arcus KH} \\ \text{comp. HE i. ang. ad C} \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{ad FE radii} \\ 10000000 \end{array} \right\} \left. \begin{array}{l} \text{Ita HG tangens arcus} \\ \text{HB i. complementi} \\ \text{basis BC} 5773502 \end{array} \right\} \left. \begin{array}{l} \text{ad ED tangentem arcus} \\ EA i. compl. lateris AC \\ 6666665 \cdot \text{par. 33} 41' 25". \end{array} \right\}$$

Ergo ipsum latus AC est partium 56 18' 35". quadrante minus per 9 & 10 hujus. Nam propter angulum ad C acutum, latus AB quadrante minus est : propter basim vero etiam quadrante minus, reliquum latus AC quadrante minus est.

Vel per 17 & 20 primi hujus,

$$\left. \begin{array}{l} \text{Vt secans anguli ad C} \\ 11547004 \end{array} \right\} \left. \begin{array}{l} \text{ad radium } 10000000 \\ \text{ad sinum com. ang. ad C} \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{Ita tangens basis} \\ 17320508 \end{array} \right\} \left. \begin{array}{l} \text{ad tangentem lateris AC angulo} \\ \text{dato adjacentis } 15000000 \text{ partium} \\ 56 18' 35". \end{array} \right\}$$

Vel per 17 primi hujus,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad sinum com. ang. ad C} \\ 8660254 \end{array} \right\} \left. \begin{array}{l} \text{ad secantem anguli ad C} \\ 17320508 \end{array} \right\} \left. \begin{array}{l} \text{ad tangentem lateris AC ang. dato} \\ \text{adjacentis } 15000000 \text{ partium} \\ 56 18' 35". \end{array} \right\}$$

Vel,

$$\left. \begin{array}{l} \text{Vt radius } 10000000 \\ \text{ad secantem anguli ad C} \\ 11547004 \end{array} \right\} \left. \begin{array}{l} \text{Ita tangens complem. basis} \\ 5773502 \end{array} \right\} \left. \begin{array}{l} \text{ad tangentem complem.} \\ \text{lateris AC } 6666665. \\ \text{ut supra.} \end{array} \right\}$$

Quinto, dato latere & angulo adjacente, invenitur basis. Radius enim est ad sinum complementi anguli : ut tangens complementi lateris ad tangentem complementi basis. Aut, radius est ad secantem anguli ; ut tangens lateris ad tangentem basis. Vel, sinus complementi anguli est ad radium , ut tangens lateris ad tangentem basis. Vel, secans anguli est ad radium ; ut tangens complementi lateris ad tangentem complementi basis .

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Maneat & hic possumus nostrum diagramma, daturque in Triangulo ABC, latus AC partium 56 18' 35": angulusque ad C, part. 30. invenietur basis BC partium 60. Nam per quartam sexti & decimam partem septimi Euclidis,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt F E radius} \\ 10000000 \end{array} \right\} \begin{array}{l} \text{ad IH sinum arcus} \\ \text{KH .i. compl.} \\ \text{HE, vel ang. ad} \\ \text{C, 8660254} \end{array} \left. \begin{array}{l} \text{Ita tangens ED .i. compl.} \\ \text{lateris AC, 6666665} \end{array} \right\} \begin{array}{l} \text{ad HG, tangentem} \\ \text{HB, compl. ba-} \\ \text{sis, 5773502. par-} \\ \text{tium 30.} \end{array} \end{array}$$

Ergo basis est partium 60, quadrante minor per 10 bujus, quia utrumque latus singulatim quadrante minus est: AC quidem ex ibi, AB vero propter angulum ad C acutum.

Vel per 17 & 20 Theorema primi bujus,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt radius 10000000} \end{array} \right\} \begin{array}{l} \text{ad secantem ang. ad} \\ \text{C 11547004} \end{array} \left. \begin{array}{l} \text{Ita tangens lateris AC} \\ 15000000 \end{array} \right\} \begin{array}{l} \text{ad tangentem basis} \\ 17320508. pa. 60. \end{array} \end{array}$$

Vel per 17 primi bujus,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt sinus compl. ang. ad} \\ \text{C 8660254} \end{array} \right\} \begin{array}{l} \text{ad radium 10000000} \end{array} \left. \begin{array}{l} \text{Ita tangens lateris} \\ \text{AC 15000000} \end{array} \right\} \begin{array}{l} \text{ad tangentem basis} \\ 17320508, pa. 60. \end{array} \end{array}$$

Vel per eidem Theorema,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt secans ang. ad} \\ \text{C 11547004} \end{array} \right\} \begin{array}{l} \text{ad radium 10000000} \end{array} \left. \begin{array}{l} \text{Ita tangens complementi} \\ \text{lateris AC 6666665} \end{array} \right\} \begin{array}{l} \text{ad tangentem complem. basi-} \\ \text{s 5773502, ut supra.} \end{array} \end{array}$$

Sexto, data basi & latere, manifestatur angulus adjacens. Tangens enim complementi lateris dati est ad radium; ut tangens complementi basis ad sinum complementi anguli adjacentis. Vel, tangens lateris dati est ad radium; ut tangens basis ad secantem anguli adjacentis. Vel, radius est ad tangentem complementi lateris dati; ut tangens basis ad secantem anguli adjacentis. Vel, radius est ad tangentem lateris dati; ut tangens complementi basis, ad sinum complementi anguli adjacentis.

Repetita & hic superiori Trianguli nostri figura, detur BC basis partium 60: latusque AC partium 56 18' 35. invenietur angulus ad C adjacens partium 30. Nam per 4 sexti & 19 septimi Euclidis,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt ED tangens arcus} \\ \text{EA .i. comp. lateris} \end{array} \right\} \begin{array}{l} \text{ad FE radius} \\ 10000000 \end{array} \left. \begin{array}{l} \text{Ita GH tangens arcus} \\ \text{HB .i. compl. basis} \\ \text{BC 5773502} \end{array} \right\} \begin{array}{l} \text{ad IH sinu arcus KH .i.} \\ \text{comp. HE vel anguli ad} \\ \text{C 8660254, par. 60.} \end{array} \end{array}$$

Ergo ipse angulus ad C est partium 30, acutus: basis enim CB est minor quadrante. Itaque per 10 bujus, utrumque latus AD & BD est quadrante circuli minus vel inajus. Sed AD unum latus est quadrante minus ex ibi. itaque & reliquum BD: prouinde oppositus angulus ad A per 9 bujus acutus est.

Aliorū per 17 & 20 primi bujus,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt tangens la-} \\ \text{teris AC} \end{array} \right\} \begin{array}{l} \text{ad radium 10000000} \\ 15000000 \end{array} \left. \begin{array}{l} \text{Ita tangens basis} \\ 17320508 \end{array} \right\} \begin{array}{l} \text{ad secantem anguli ad C} \\ \text{adjacentis 11547002} \\ \text{partium 30.} \end{array} \end{array}$$

Vel per 17 primi bujus,

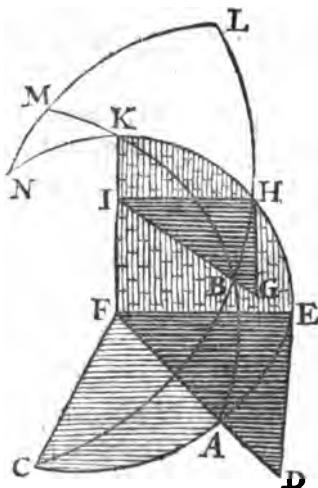
$$\begin{array}{l} \left. \begin{array}{l} \text{Vt radius 10000000} \end{array} \right\} \begin{array}{l} \text{ad tangentem comp. late-} \\ \text{ris AC 6666665} \end{array} \left. \begin{array}{l} \text{Ita tangens basis} \\ 17320508 \end{array} \right\} \begin{array}{l} \text{ad secantem ang. ad C} \\ 11547002, part. 30. \end{array} \end{array}$$

Vel,

$$\begin{array}{l} \left. \begin{array}{l} \text{Vt radius 10000000} \end{array} \right\} \begin{array}{l} \text{ad tangentem lateris} \\ \text{AC 15000000} \end{array} \left. \begin{array}{l} \text{Ita tangens comp. basi-} \\ \text{s 5773502} \end{array} \right\} \begin{array}{l} \text{ad sinum compl. ang.} \\ \text{ad C 8660254.} \end{array} \end{array}$$

Sep-

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Septimo, data basi & angulo obliquo alterutro, inventur reliquus. Nam ut sinus complementi basis est ad radium: ita tangens complementi anguli, ad tangentem anguli reliqui. Aut, ut secans basis est ad radium; ita tangens anguli, ad tangentem complementi reliqui. Vel, ut radius est ad sinum complementi basis; ita tangens anguli, ad tangentem complementi reliqui. Vel, ut radius est ad secantem basis; ita tangens complementi anguli, ad tangentem reliqui.

Detur in Triangulo A B C, basis B C partium 60: & angulus ad C partium 30: dabitur reliquus ad B partium 73 53' 52". Repetitur enim figura, qua fuit sexto porismate Theorema præmissi. Demonstratum fuit, illuc, arcum H L aequalem esse basi B C, & mensuram esse anguli ad N, in Triangulo N M K rectangulo ad M: Item E H mensuram anguli ad C in Triangulo A B C, aequalem esse basi N K in Triangulo N M K; M L vero arcum, mensuram esse anguli ad B quæfisi, & M N complementum ejusdem. Quare cum in Triangulo N M K, detur angulus ad N, cum basi N K: datur etiam per quartum hujus NM, latus angulo adjacens, i.e. complementum anguli ad B quæfisi. Nam,

$$\begin{array}{l} \text{Vt sinus comp. ang.} \\ \text{ad N i. basis data} \\ 5000000 \end{array} \left\{ \begin{array}{l} \text{ad radium} \\ 10000000 \end{array} \right\} \begin{array}{l} \text{Ita tangens comp.} \\ \text{bas N K i. ang.} \\ \text{ad C } 17320508 \end{array} \left\{ \begin{array}{l} \text{ad tangentem comp. M N i. ad tan-} \\ \text{gentem arcus M L vel ang. ad B} \\ \text{ad C } 34641016 \text{ part. } 73 53' 52'' \text{ acuti.} \end{array} \right\}$$

Nam quia basis quadrante minor est, latera sunt quadrante majora, vel minora similiiter per 10 hujus. Sed A B latus quadrante minus est per 9 hujus; propter angulum ad C oppositum acutum: Ergo & reliquum latus quadrante minus est, & reliquus angulus acutus. Alter per 17 vel 20 primi hujus,

$$\begin{array}{l} \text{Vt secans basis} \\ 20000000 \end{array} \left\{ \begin{array}{l} \text{ad radium } 10000000 \end{array} \right\} \begin{array}{l} \text{Ita tang. an-} \\ \text{guli ad C} \\ 5773502 \end{array} \left\{ \begin{array}{l} \text{ad tangentem complementi anguli} \\ \text{reliqui } 2886751 \text{ partium} \\ \text{16 } 6' 8'' \end{array} \right\}$$

Vel per 17 primi hujus,

$$\begin{array}{l} \text{Vt radius } 10000000 \end{array} \left\{ \begin{array}{l} \text{ad finum compl. basis} \\ 5000000 \end{array} \right\} \begin{array}{l} \text{Ita tang. ang. ad C} \\ 5773502 \end{array} \left\{ \begin{array}{l} \text{ad tang. compl. anguli} \\ \text{reliqui } 2886751. \end{array} \right\}$$

Vel per idem Theorema,

$$\begin{array}{l} \text{Vt radius } 10000000 \end{array} \left\{ \begin{array}{l} \text{ad secantem basis} \\ 20000000 \end{array} \right\} \begin{array}{l} \text{Ita tang. compl. ang.} \\ \text{ad C } 17320508 \end{array} \left\{ \begin{array}{l} \text{ad tangent. ang. reliqui} \\ 34641016. \end{array} \right\}$$

Postremo, dato utroque angulo obliquo datur basis. Tangens enim anguli alterutrius est ad radium; ut tangens complementi anguli reliqui, ad finum complementi basis. Vel, tangens complementi anguli alterutrius est ad radium; ut tangens anguli reliqui ad secantem basis. Aut, radius est ad tangentem anguli alterutrius; ut tangens anguli reliqui ad secantem basis. Aut, radius est ad tangentem complementi anguli alterutrius; ut tangens complementi anguli reliqui ad finum complementi basis.

Manente superiori diagrapha, detur angulus ad C part. 30: & reliquus ad B partium 73 53' 52". Dabitur basis B C partium 60. Assumatur enim & bic Triangulum N M K rectangulum: in quo cum detur latus N M, complementum scilicet arcus M L i.e. anguli ad B; & basis N K, equalis arcui H E, i.e. angulo reliquo ad C, datur etiam angulus ad N, vel arcus L H i.e. basis B C. Nam per 6 porisma hujus,

$$\begin{array}{l} \text{Vt tangens compl. N M} \\ \text{i.e. arcus M L vel angul.} \\ \text{ad B, } 34641016 \end{array} \left\{ \begin{array}{l} \text{ad radium } 10000000 \end{array} \right\} \begin{array}{l} \text{Ita tangens comp. basis} \\ \text{N K, i.e. arcus H E vel} \\ \text{ang. ad C } 17320508 \end{array} \left\{ \begin{array}{l} \text{ad finum compl. ang. ad N} \\ \text{i.e. arcus L H vel basis B C} \\ \text{500000 partium 30.} \end{array} \right\}$$

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Ergo basis est partium 60, quadrante minor per 11 hujus, quia angulus uterque acutus est.

Aliet per 17 & 20 Theorema primi hujus,

$$\begin{array}{l} \text{Vt tangens comp. ang. ad B} \\ 2886751 \end{array} \left\{ \begin{array}{l} \text{ad radium} \\ 10000000 \end{array} \right\} \left\{ \begin{array}{l} \text{Ita tang. ang. ad C} \\ 5773502 \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem basis} \\ \text{partium 60.} \end{array} \right\}$$

Vel per 17 primi hujus,

$$\begin{array}{l} \text{Vt radius 10000000} \\ \text{B 2886751} \end{array} \left\{ \begin{array}{l} \text{ad tang. ang. ad B} \\ 34641016 \end{array} \right\} \left\{ \begin{array}{l} \text{Ita tangens anguli ad C} \\ 5773502 \end{array} \right\} \left\{ \begin{array}{l} \text{ad secantem basis} \\ 20000000 \end{array} \right\}$$

Vel per idem Theorema,

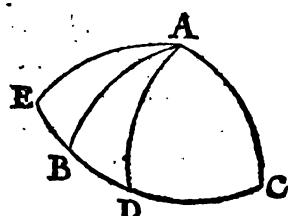
$$\begin{array}{l} \text{Vt radius 10000000} \\ \text{B 2886751} \end{array} \left\{ \begin{array}{l} \text{ad tang. comp. ang. ad} \\ \text{ad C 17320508} \end{array} \right\} \left\{ \begin{array}{l} \text{Ita tang. comp. ang.} \\ \text{su 5000000} \end{array} \right\}$$

Atque ita calculus rectangulorum Triangulorum expositus est. Sequitur

Obliquangulorum Sphericorum Calculus.

14. Triangulum obliquangulum Sphericum est, cuius tres anguli obliqui sunt.

15. Si triangulum obliquangulum, acutos duos angulos aut obtusos habuerit, perpendicularis arcus, ab angulari punto tertii egressiens, cadit intra triangulum: sin angulorum alter acutus, & reliquus obtusus extiterit, cadit extra.

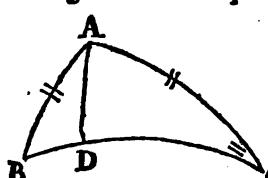


Esto obliquangulum Triangulum A B C, acutangulum ad B & C: dico perpendiculararem A D, demissam ab A vertice anguli tertii, cadere intra Triangulum. Nam si non cadit intra: vel lateri alterutri coincidat, vel extra cadat necesse est. Si lateri alterutri coincidat: tunc angulus ad C, vel B rectus est, quod est contra thesin. Si extra cadit exempli gratia in E: angulus ad E rectus est. Sed angulus A B E obtusus est, reliquis scilicet acutis A B C. Itaque per 9 hujus, latus A E est majus circuli quadrante. Rursus quia angulus ad C acutus est in Triangulo A E C rectangulo, per citatum theorema, latus A E quadrante minus est. Itaque A E latus, commune utriusque Triangulo A E B, & A E C, est quadrante majus & minus; quod absurdum est. Consequitur igitur perpendiculararem cadere intra Triangulum datum.

Esto verò A E B triangulum, obtusangulum ad B: acutangulum ad E. Dico A D perpendiculararem cadere extra Triangulum, in latus E B continuatum. Secus si non: vel lateri alterutri coincidit, vel intra cadit. Sed coincidere nequit, quia tunc alterat angulorum ad B, vel E rectus esset: Intra cadere nequit, quia uterque angulorum ad B, & E, acutus esset, vel obtusus, ex prima parte hujus. Virumque est contra thesin. Consequitur igitur, perpendiculararem extra Triangulum cadere, si alter angulorum acutus, & reliquus obtusus extiterit: que fuerunt demonstranda.

Π Ο Ρ Ι Σ Μ Α Τ Α quatuor.

Primo itaque in Triangulo obliquangulo datis duobus lateribus & angulo unius eorum opposito, insuper nota specie anguli alteri dato lateri oppositi, anguli reliqui latusque tertium inveniuntur. Demissus enim ab angulo datis lateribus contento, in oppositum latus (continuum si oportet) perpendicularis arcus, obliquangulum Triangulum, in duo rectangula secat, ex quorum calculo quæsita inveniuntur.



Esto Sphericum Triangulum A B C obliquangulum: in quo dentur latera, A C part. 50, A B part. 26 22' 20'', & angulus ad C part. 30, cum specie anguli ad B acuta; dabuntur anguli ad A & B, cum tertio latere B C. Descendet enim perpendicularis A D in latus B C, qui intra Triangulum cadit, propter utrumque angulum ad B & C acutum; fixante rectangula Triangula duo, A D C & A D B, daturque in Triangulo A D C basis A C part. 50, & angulus ad C part. 30. Itaque per primum per ipsum duodecimi hujus, A D est part. 22 31' 15'': quadrante circuli minus, per 9 hujus, quia oppositus angulus acutus est.

L

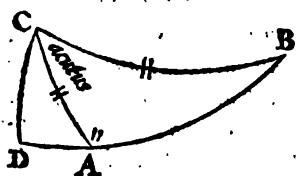
Iam

Geometriæ Triangulorum Liber IIII.

Iam si quadratus angulus ad B, dabitur ad minimus perpendiculus A D inventus. Nam in Triangulo A D B rectangulo, datus basis A B part. 26 22' 20", & latus A D part. 22 31' 15". Itaque per 2 porisma 12 bujus, angulus ad B est part. 59 34' 21" acutus per 9 bujus, quia oppositum latus quadrante minus est.

Eodem modo definitur angulus ad A. Nam in Triangulo A D B datur basis A B part. 26 22' 20", & latus A D part. 22 31' 15". Itaque per 6 porisma 13 bujus, angulus B A D est part. 33 14' 53", acutus. Nam quia basis A B est minor quadrante, utrumque latus A D & B D, quadrante minus est, aut maius. Sed A D latus minus est quadrante, itaque & B D. Quare per 9 bujus, angulus ad A oppositus acutus est: Rursus in Triangulo rectangulo A D C, datur basis A C part. 50, & angulus ad C part. 30, & latus A D part. 22 31' 15"; Itaque per 7 porisma 12, vel per 6, aut 7 decimiertii (plura enim hic data sunt) angulus D A C est part. 69 38' 20", acutus, quia basis & latus quadrante minor est. Anguli vero B A D, & C A D equeles sunt angulo B A C, ergo angulus B A C est part. 102 53' 13".

Quin & latus B C eadem methodo investigatur. In Triangulo enim rectangulo B A D datur basis A B part. 26 22' 20", & latus A D part. 22 31' 15". Quare per 4 porisma duodecimi bujus, B D est part. 14 5' 44", quadrante minor per 10 bujus, quia basis cum dato latere figillatim quadrante minores sunt. Præterea in Triangulo rectangulo A D C datur basis A C part. 50, angulus ad C partium 30, cum latere A D part. 22 31' 15". Quare per 4 porisma 12 bujus, vel per 2 aut 4 decimiertii, latus D C est part. 45 34' 16", quadrante minus, quia basis & latus datum quadrante minus est. Iam cum B D sit part. 14 5' 44"; & D C part. 45 34' 16", latus B D C, utriusque summa est part. 60.



Et sic quidem propositi Trianguli postulata innescunt, perpendiculari intra Triangulum cadente: Diversa autem parum est ratio, perpendiculari cadente extra. Esto enim obliquangulum Triangulum A B C, in quo detur A C, latus part. 26 22' 20", B C partium 60, & angulus ad B part. 102 53' 13", cum specie anguli ad C acuta: Invenientur reliqui anguli ad C & B, cum latere tertio A B. Emisso enim perpendiculari arcu C D, ex angulari punto datorum laterum C, cadente extra, proper angulos ad A & C specie diversos, sicut supra duo Triangula rectangula A D C & B D C, ex quorum calculo quesita inveniuntur. In Triangulo enim A D C datur basis A C part. 26 22' 20", cum angulo ad A part. 77 6' 47", reliquo scilicet ipsius B A C ad semicirculum; itaque C D perpendicularis per primum porisma 12 bujus est part. 25 39' 32".

Secundo, in Triangulo B D C rectangulo datur basis B C part. 60, & latus C D part. 25 39' 32"; Itaque per secundum porisma 12 bujus, angulus ad B est part. 30.

Tertio, in eodem Triangulo rectangulo B D C, datur basis B C part. 60, cum latere C D part. 25 39' 32", ergo per 6 porisma 13 bujus, angulus B C D est part. 73 53' 52". Item in Triangulo rectangulo A D C, datur basis A C part. 26 22' 20", cum latere C D part. 25 39' 32"; ergo per idem porisma angulus A C D est part. 14 19' 31". Ablato vero angulo A C D ex angulo B C D, relinquitur angulus A C B partium 59 34' 21".

Postremo, in Triangulo rectangulo B D C, datur basis B C part. 60, cum angulo ad C part. 73 53' 52", ergo per primum porisma 12 bujus, latus angulo dato oppositum B D est part. 56 18' 35". Item in Triangulo rectangulo A D C datur basis A C part. 26 22' 20", cum angulo ad C part. 14 19' 31", ergo per idem porisma, latus A D angulo C oppositum est part. 6 18' 35". Anfer autem A D ex B D, & relinquatur latus A B part. 50. Que fuerum investiganda.

Secundo, datis duobus angulis, & latere uni eorum opposito, patescunt reliqua latera, & angulus tertius, si modo constiterit utrum latus ignotum dato angulo oppositum, quadrante maius fuerit an minus. Perpendicularis enim arcus eductus à termino lateris dati, in latus utrique angulo dato adjacens, (continuatum si oportet) obliquangulum in duo Triangula rectangula dividit, unde postulata innescunt.

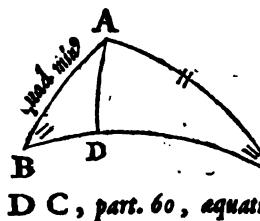
Detur A B C triangulum non rectangulum, & in eo latus A C part. 50, cum angulis, ad C quidem part. 30, sed ad B part. 59 34' 21": siue A B latus ignotum quadrante minus. Innescent hinc reliqua latera A B & D C, cum angulo tertio ad A. Primum enim in Triangulo rectangulo A D C datur basis A C part. 50, cum ang. ad C part. 30. Ergo per primum porisma 12 bu-

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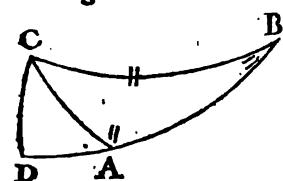
bujus perpendicularis AD est part. 22 31' 15": cadisque intra Triangulum, quia B & C anguli dati sunt acuti.

Secundo, in Triangulo rectangulo ADB datur latus AD part. 22 31' 15", cum opposito angulo B part. 59 34' 21"; itaque basis AB , per 3 porisma 12 bujus, est part. 26 22' 20", quadrante minor ex ibef.



Tertio, in Triangulo ADB rectangulo, ex latere AD part. 22 31' 15", & angulo B part. 59 34' 21", datur latus BD per idem porisma partium 14 5' 44", quadrante minus, propter AB basin quadrante minorem. Item in Triangulo ADC rectangulo, ex latere AD part. 22 31' 15", & ang. C part. 30 (vel ex aliis, quia plura data sunt) datur latus DC part. 45 54' 16". Summa vero laterum BD & DC , part. 60, aquatur lateri BC .

Postremo, in Triangulo rectangulo ADC , propter datum basim AC , cum latere DC , & angulo C , invenitur angulus DAC partium 69 38' 20". Item in Triangulo rectangulo ADB , ex data basi AB , & latere BD , cum angulo B , patescit multis modis ang. BAD part. 33 14' 53". Summa vero angulorum DAC & CAD , equalis est angulo BAC tertio, part. 102 53' 13".



Et sic postulata porismata nostri investigata sunt, perpendiculari arcu cadente intra Triangulum. Similis fere est ratio si cadat extra. Detur enim in apposito Triangulo ABC obliquangulo, angulus ad A part. 102 53' 13", ad B part. 30, cum latere BC part. 60; innotescunt hinc reliqua latera & angulus tertius.

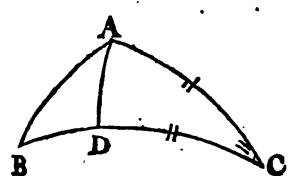
Primum enim, quia perpendicularis CD cadit extra, datur in Triangulo rectangulo BDC , basi BC partium 60, cum angulo C part. 30. Quia per primum porisma 12 bujus, perpendicularis CD est partium 25 39' 32".

Secundo, in Triangulo rectangulo ACD , datur perpendicularis CD part. 25 39' 32", cum angulo ad A , residuo scilicet ipsius BAC ad semicirculum part. 77 6' 47"; Ergo per secundum porisma 13 bujus, angulus ACD est part. 14 19' 31". Item in Triangulo rectangulo BDC , datur perpendicularis CD part. 25 39' 32", & angulus ad B part. 30. Ergo per idem porisma, vel per alia quia plura data sunt, angulus BDC est part. 73 53' 52". Ausfer autem angulum ACD ex angulo BCD , & reliquas erit angulus tertius ACB part. 59 34' 21".

Tertio, in Triangulo rectangulo ADC , ex dato utroque angulo C & A cum latere CD , datur reliquum latus DA part. 6 18' 35". Item in Triangulo rectangulo BDC , ex dato utroque angulo B & C , etiam latere CD , & basi BC , multis modis manifestatur latus BD part. 56 18' 35". Tolle autem latus DA part. 6 18' 35", ex latere BD part. 56 18' 35", & remanebit latus AB part. 50.

Postremo, in Triangulo rectangulo ADC ex dato utroque angulo C & A , atque etiam utroque latere CD & AD , variis modis patescit basis AC part. 26 22' 20"; Quae fuerint investiganda.

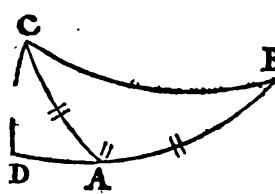
Tertio, datis duobus lateribus, & angulo ab iis comprehenso, tertium latus, & anguli reliqui innotescunt. Perpendicularis enim arcus, à termino lateris alterutrius dati, in reliquum datum (si necesse sit productum) emisus, obliquangulum triangulum in duo rectangula partitur, ex quorum calculo ignota manifestantur.



Esto obliquangulum Triangulum ABC , in quo dentur latera AC part. 50, BC part. 60, cum angulo ad C ab iis comprehenso part. 30. Perpendicularis AD ut supra invenitur part. 22 31' 15", cadisque intra Triangulum, ut calculus docebit. Latus enim CD in Triangulo rectangulo ADC , invenitur per 4 porisma 12 bujus, vel per alia, quia plura data sunt, part. 45 54' 16", minus latere BC part. 60.

Itaque BD est part. 14 5' 44", & perpendicularis AD intra Triangulum cadit. Porro ex lateribus AD & BD in Triangulo rectangulo ADB cognito, invenitur basis AB , per 5 porisma duodecimi bujus, part. 26 22' 20": Item angulus ad B , per tertium porisma decimiorum bujus, vel per alia, quia plura data sunt, partium 59 34' 21". Postremo, angulus BAD in eodem Triangulo ADB ; invenitur part. 33 14' 53"; & angulus DAC in Triangulo ADC part. 69 38' 20". Ergo angulus BAC sumusque summa est part. 102 53' 13".

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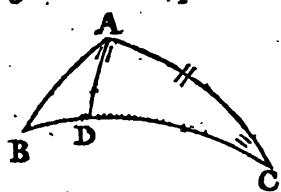


Dantur vero in Triangulo obliquangulo A B C apposita latera, A B partium 50, A C part. 26 22' 20", cum angulo A inclusu part. 102 53' 13"; perpendicularis D C erit part. 25 39' 32", ut supra, quadrante minor. Nam angulus C A D est acutus, residuus scilicet C A B obtusus, & basis A C est minor quadrante. Itaque perpendicularis arcus C D cadit extra. Dantur autem in Triangulo A D C rectangulo latus C D part. 25 39' 32", & angulus ad A part. 77 6' 47", reliquus, scilicet angulus C A B, ad semicirculum: ergo latus D A est part. 6 18' 35". A B vero est part. 50: totus igitur arcus D A B est part. 56 18' 35".

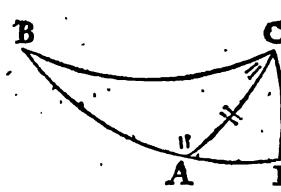
Secundo, in Triangulo B D C rectangulo dantur latera, C D part. 25 39' 32", & D B part. 56 18' 35": ergo basis B C invenitur part. 60, angulus ad C part. 30, & angulus B C D part. 73 53' 52".

Tandem in Triangulo A D C rectangulo, reperitur angulus A C D partium 14 19' 31", qui subductus ex angulo B C D part. 73 53' 52", relinquit angulum A C B part. 59 34' 21". Quae fuerunt indaganda.

Postremo datis duobus angulis, una cum latere utrique adjacente, reliqua latera, & angulus tertius investigantur. Perpendicularis enim arcus ab angulo alterutro in oppositum latus (continuum si oportet) egrediens, obliquangulum Triangulum in duo rectangula secat, ex quorum calculo quæsita dantur.

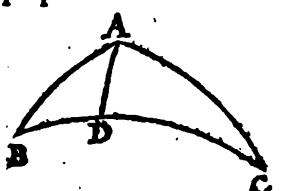


Este Triangulum ABC non rectangulum, sique angulus ad A part. 102 53' 13", ad C part. 30, & latus A C part. 50. Erit A D part. 22 31' 15", latus scilicet Trianguli rectanguli A D C: & angulus C A D part. 69 38' 20", minor angulo B A C dato; ergo reliquus B A D est partium 33 14' 53", & proin perpendicularis intra Triangulum cadit. Hinc in Triangulo A D B invenitur latus A B (ex dato latere A D, cum angulo ad A) part. 26 22' 20": item angulus tertius ad B part. 59 34' 21", cum latere B D, part. 14 5' 44". Latus vero D C invenitur in Triangulo A D C, part. 45 54' 16". Ergo totum latus B D C est part. 60.



Sit vero angulus ad A in Triangulo apposito A B C part. 102 53' 13", ad C part. 59 34' 21", & latus A C part. 26 22' 20": invenietur C D perpendicularis part. 25 39' 32", quadrante minor; ergo angulus ad B, in Triangulo rectangulo B D C, per 9 bujus acutus est, & perpendicularis cadit extra; anguli enim ad A & B specie diversi sunt. Hinc reperiuntur, primum in Triangulo A D C, latus D A part. 6 18' 35", & in Triangulo C D B, latus D B part. 56 18' 35". Auso autem D A ex D B, & reliquum erit latus A B part. 50. Adhuc in eodem triangulo C D B, invenitur angulus tertius ad B part. 30, & latus B C part. 60. Quae fuerunt indaganda.

16. In obliquangulo Triangulo sinus angulorum sinibus oppositorum laterum directe proportionales sunt.



Este ut supra obliquangulum Triangulum A B C, secum per A D perpendiculararem, in duo Triangula rectangula A D C & A D B; ita sinus anguli B esse ad sinus lateris oppositus A C, ut sinus anguli C ad sinus oppositi lateris A B. Nam per 7 porisma 12 bujus est,

VI sinus ang. B, ad sinus lateris A D, ita sinus ang. D, ad sinus lat. A B.

Item ut sinus ang. C ad sinus lateris A D, ita sinus ang. D, ad sinus lateris A C.

Atque per 19 Septimi Euclidis, factus à sinus A D in sinus ang. D equatur factio à sinus B in sinus A B, & facto à sinus C in sinus A C. Itaque per eandem,

VII sinus ang. B ad sinus oppositi lateris A C, ita sinus ang. C ad sinus oppositi lateris A C. Eademque est ratio in reliquo angulo A, & opposito latere B C. Quid etas demonstrandum.

PROPOSITA duo.

Primum igitur datis duobus lateribus, cum angulo uni datorum laterum opposito,

ma-

Geometriæ Triangulorum Liber I I I I.

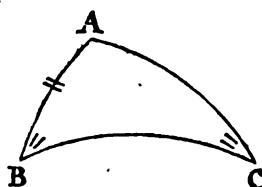
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manifestatur angulus, alteri datorum laterum oppositus. Est enim ut sinus lateris dati ad sinum anguli oppositi; ita sinus lateris alterius dati, ad sinum anguli oppositi.

In exemplo dentur in obliquangulo Triangulo A B C apposito duo Latera, A B part. 26 22' 20", A C part. 50, cum angulo ad C partium 30. Invenietur angulus ad B partium 59 34' 21". Nam

Vt sinus lateris A B 4442009 ad sinum anguli oppositi C 5000000, ita sinus lateris B C 7660445 ad sinum anguli oppositi D 8622725, partium 59 34' 21".

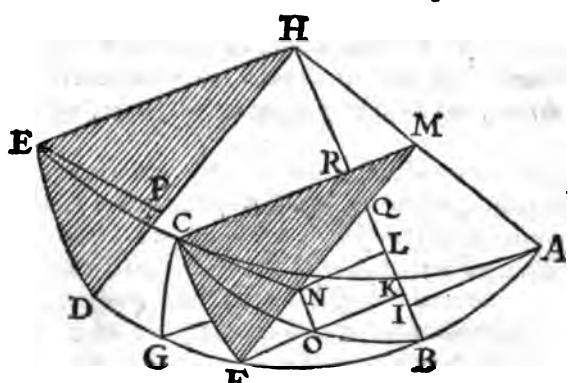
Secundo, datis duobus angulis, cum latere uni datorum angulorum opposito, invenitur latus alteri datorum angulorum oppositus. Nam ut sinus anguli dati ad sinum lateris oppositi, ita sinus alterius anguli dati, ad sinum lateris oppositi.



Exempli causa, dentur in Triangulo obliquangulo A B C duo anguli, unus ad C partium 30, alter ad B part. 59 34' 21", cum latere A B part. 26 22' 20": Invenietur A C latus part. 50. Nam

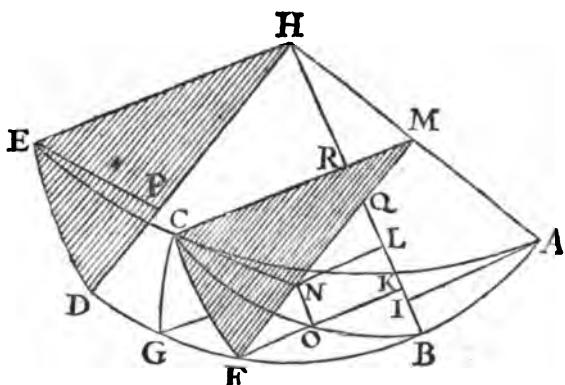
Vt sinus anguli C 5000000, ad sinum A B lateris oppositi 4442009: Ita sinus anguli B 8622725, ad sinum A C lateris oppositi 7660445 part. 50, ut supra.

17. In obliquangulo triangulo, quadratum radii est ad planum sinuum rectorum laterum duorum, ut sinus versus anguli ab iisdem comprehensu, ad differentiam sinuum versorum tertii lateris, & reliquorum laterum differentia. Quadratum autem radii est ad planum sinuum rectorum angulorum duorum, ut sinus versus lateris, utrique angulo adjacentis, ad differentiam sinuum versorum tertii anguli, & differentia datorum angulorum unius, & alterius ad semicirculum complementi.



Hoc Theorema verum est in omni Triangulo, tum rectangulo, tum obliquangulo, verum quia usus ejus potissimum est in Triangulis obliquangulis, ideo hic de obliquangulis ratiōne enumeratur. Sit igitur Sphericum Triangulum A B C obliquangulum, cuius latera A B & A C inegalitia, & signatim quadrante circuli minora, producantur in E & D, ut A C E & A B D quadrantes sint maximorum circulorum. Facto vero A polo, describatur arcus D E intervallo A D; & arcus C F intervallo A F; erit tunc arcus D E per 8 bujus mensura anguli ad A; arcus vero A F aequalis erit arcui A C. Item polo B, & distantia B C describatur arcus C G, qui aequalis erit arcui B C; & proinde arcus B F differentia erit laterum A C & A B, & arcus G F differentia tertii lateris B C, & reliquorum laterum differentia B F. Emittantur deinde ex H communis centro quadratum A D & A F, semidiametri H A, H B, & H D, in puncta A, B, D; & à terminis arcum A B, B F, & B G, demittantur perpendicularares A I, F K, & G L, in semidiametrum H B; erunt haec arcum dictorum recti sinus, per 7 primi bujus; B I autem, B K, & B L, versi sinus corundem per 10 ejusdem: & proinde K L differentia sinuum versorum lateris B C vel B G, & reliquorum laterum differentia B F. Praterea à termino arcus A F descendat perpendicularis F M in semidiametrum H A, erit hac sinus rectus lateris A F. Vbi autem G L & F M se se intersectant punctum N, ex quo ducatur N O parallela H B; adeoque per 34 primi elementorum aequalis ipsi K L. Adhac à termino arcus D E, demittantur perpendicularis E P in semidiametrum H D, erit hac sinus rectus arcus D E; & D P sinus versus ejusdem. Postrem à communis termino arcum F C & G C ducentur recta C N in N, sectionem rectarum G L & F M; erit hac normalis rectis G N & F N. Arcus enim F C & G C per 5 bujus normales sum quadranti A B D, transeunt per A & B polos corundem. Itaque communis eorum sectio, qua per 3 undecimi Euclidis est recta linea, nempe C N, est plano quadrantis A B D normalis per 19 ejusdem. Transit autem sectio communis arcum dictorum per N punctum, ex conversione definitionis linea perpendiculariter super planum eretta. Quare C N est sinus rectus arcus F C, & F N sinus versus ejusdem.

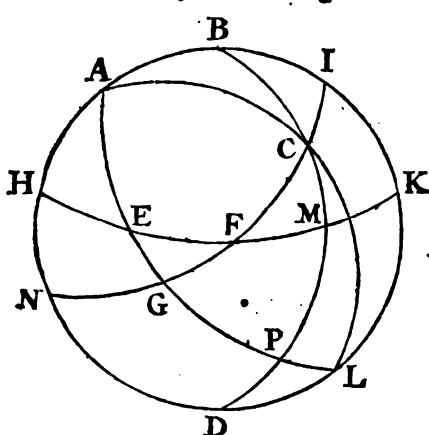
Geometriæ Triangulorum Liber IIII.



His vero in hunc modum expeditius, dico DH radium esse ad FM sinum rectum lateris AC, ut DP sinus versus anguli ad A, ad FN, sinum versus arcus FC. Item HA radium esse ad AI, sinum rectum lateris AB, ut FN sinus versus arcus FC, ad NO differentiam sinuum versorum tertii lateris, & reliquorum laterum differentia. Hoc est, per multiplicationem terminorum, quadratum radii esse ad planum sinuum rectorum FM & AI, ut DP sinus versus anguli ad A ab iisdem lateribus comprehensi, ad NO differentiam sinuum versorum tertii lateris, & reliquorum laterum differentia. Triangula enim HEP, & MCN sunt aquiangula, ob rectos angulos ad P & N, aequali ad H & M, inclinationis scilicet angulum quadrantis ACE, ad quadrantem AFD. Itaque per 4 Sexti Euclidis, latera habent proportionalia. Quare ut EH ad CM, ita PH ad NM. Et quia DH ex fabrica equatur ipsi DH, & FM ipsi CM, DH est ad FM, ut PH ad NM: adeoque per 5 Quinti Euclidis, ut DH ad FM, ita DP ad FN. Secundo Triangula FON & FKQ & HMQ sunt aquiangula, ob rectos angulos ad O & K, communem ad F. Item triangula HMQ & HAI sunt aquiangula, ob rectos angulos ad M & I, communem ad H; itaque per 4 Sexti elementorum HA est ad AI, ut FN ad NO. Quod erat demonstrandum.

Ita vero patet veritas prima partis Theorematis hujus. Est enim Triangulum propositum, laterum sit quadrante circuli minorum, valet tamen superior ratiocinatio in Triangulis, quorum latera comprehendentia angulum, vel quadrante circuli majora sunt, vel unum majus, alterum minus. Nam ex 7 primis hujus, sinus rectus duabus peripheriis communis est, uni, circuli quadrante minori; alteri, quadrante circuli majori. Imo si latera equalia dentur, non absimilis est argumentandi forma, nisi quod NO tunc sit tertii lateris sinus versus.

Secunda porro pars Theorematatis, quam jure Nobis vendicamus, quod à Nobis primum inventa sit, eodem modo demonstratur quo prima, si prius novum describatur Triangulum, per polos laterum Trianguli dati. Hujus enim latera angulis, & anguli lateribus primi Trianguli ita respondent, ut in secunda parte Theorematis eadem ferè ratione argumentari liceat, quā in prima, sicuti ex sequentibus evadet manifestū.



Sit enim Triangulum ABC idem quod supra, obtusangulum scilicet ad B, acutangulum ad A & C; & producatur ipsius latus minimum AB ex polo F in circulum AKDA: reliqua vero latera producantur in semicirculos, AC quidem ex polo G in semicirculum ACL, BC autem ex polo E in semicirculum BCD. Describatur quoque ex polo A semicirculus NFI transiens per polos G & F; & ex polo B semicirculus HFK, transiens per polos E & F; tandemque ex polo C semicirculus AGL, transiens per polos E & G; habebimus tunc novum Triangulum Sphaericum EFG, cuius tria latera respondebunt tribus angulis Trianguli ABC; & hujus tria latera respondebunt tribus angulis Trianguli EGF. Nam quod ad latera Trianguli EGF attinet, primum latus EF aequaliter est residuo anguli ABC ad semicirculum. Nam E est polus semicirculi BMD, & F est polus semicirculi BKD, & proinde EM & FK sunt circuli quadrantes. Ablato igitur communi medio FM, relinquuntur arcus EF & MK aequales. Atqui MK subtendit angulum MBK per 8 hujus, hoc est residuum anguli ABC ad semicirculum. Itaque latus EF est aequaliter residuo anguli ABC ad semicirculum.

Secundo, latus GF aequaliter est angulo BAC. Nam F est polus semicirculi BKD, & G est polus semicirculi ACL: ideoque GC & FI sunt circuli quadrantes. Remoto igitur communi medio FC, remanent arcus GF & CI aequales. Sed CI est mensura anguli BAC, per 8 hujus. Ergo latus GF est aequaliter angulo BAC.

Tertio, latus GE est aequaliter angulo ACB. Nam G est polus semicirculi ACL, & E est polus semi-

Geometriæ Triangulorum Liber IIII.

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Semicirculi BMD: itaque EP & GL sunt circuli quadrantes. Dempto igitur communi medio GP, reliqui arcus EG & PL aquantur. Sed PL metitur angulum ad C per octavum hujus, hoc est angulum ACB. Ergo latus GE est aequalis angulo ACB.

Atque ita demonstratum est tria latera Trianguli EFG respondere tribus angulis trianguli ABC. Quod autem tres anguli trianguli EFG, respondentes tribus lateribus trianguli ABC ita ostenditur.

Primo, angulus EFG aequalis est lateri AB. Nam A est polus semicirculi NFI, & B est polus semicirculi HFK. Itaque BK & AI sunt circuli quadrantes. Quare ablato communi medio BI, residui arcus BK & AI sunt aequales. Atqui IK mensurat per 8 hujus angulum IFK, id est angulum EFG. Quare angulus EFG est aequalis lateri AB.

Secundo, angulus FEG est aequalis lateri BC. Nam B est polus semicirculi HFK, & C est polus semicirculi AGL. Quare DM & PC sunt quadrantes circuli; à quibus remoto communi medio CM, residui arcus PM & BC aquantur. At vero PM per 8 hujus est mensura anguli PEM, id est anguli FEG. Quamobrem angulus FEG est aequalis lateri BC.

Tertio, angulus EGF est aequalis complemento lateris AC. Nam B est polus semicirculi HFK, & C est polus semicirculi AGL. Itaque BG & CG sunt circuli quadrantes. Arcus autem AC, per 8 hujus metitur angulum AGC, id est EGF. Ergo angulus EGF aequalis est complemento lateris AC. Nam quia EF non metitur angulum ABC, sed residuum ad semicirculum MBK, idcirco etiam angulus G oppositus lateri EF non metitur latus AC, sed ipsius complementum ad semicirculum CL.

Apparet autem ex hac demonstratione veritas secunda partis Theorematis nostri. Nam quia latera & anguli secundi Trianguli EFG respondent angulis & lateribus Trianguli primi ABC, eo modo quo ante demonstravimus, sequitur sane ex eo, eandem esse proportionem laterum & angulorum in triangulo secundo, qua supra demonstrata est in primo. Sunt ergo termini proportionales in primo Triangulo isti

Primo, secundum demonstrationem primæ partis Theorematis.

$\frac{8}{D \text{ H radius}}$	$\frac{6}{F M \text{ sinus rectus lateris } AC}$	$\frac{4}{D P \text{ sinus versus anguli dati}}$	$\frac{3}{F N \text{ quartus}}$
$\frac{8}{A \text{ H radius}}$	$\frac{4}{A I \text{ sinus rectus lateris } AB}$	$\frac{3}{F N \text{ quartus}}$	$\frac{1\frac{1}{2}}{N O \text{ differentia sinuum versus tertii lateris &c.}}$

Secundò, per multiplicationem terminorum.

$\frac{64}{\text{Quadratum radii } DH}$	$\frac{24}{\text{Planum sinuum rectorum}}$	$\frac{4}{D P \text{ sinus versus anguli dati.}}$	$\frac{1\frac{1}{2}}{N O \text{ differentia sinuum versus tertii lateris &c.}}$
vel AH	FM & AI		

Tertiò, per terminorum transpositionem.

$\frac{8}{A \text{ H radius}}$	$\frac{6}{F M \text{ sinus rectus lateris } AC}$	$\frac{4}{A I \text{ sinus rectus lateris } AB}$	$\frac{3}{F N \text{ quartus}}$
$\frac{8}{D \text{ H radius}}$	$\frac{3}{F N \text{ quartus}}$	$\frac{4}{D P \text{ sinus versus anguli dati.}}$	$\frac{1\frac{1}{2}}{N O \text{ differentia sinuum &c.}}$

Tot modis licet variare proportionum terminos, in prima Theorematis parte. Verum quia tertius modus & facilior est, & ad usum maxime accommodatus, ideo eum ceteris præstulimus, & in sequentibus porismatis usupavimus.

II O P I S M A T A quartu.

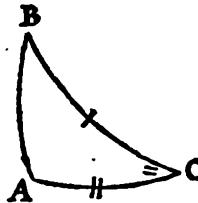
Primum itaque in obliquangulo triangulo, datis duobus lateribus & angulo ab iis comprehenso, investigatur latus tertium. Radius enim est ad sinum rectum lateris unius dati, ut sinus rectus alterius dati ad quartum. Item Radius est ad quartum, ut sinus versus anguli dati ad differentiam sinuum versorum tertii lateris, & reliquorum laterum differentiarum. Hæc igitur differentia ad sinum versus differentiarum laterum adjecta, componit sinum versus lateris quæsiti.

Repetatur penultima nostra diagrpha, & assumatur ut supra Triangulum obliquangulum Sphericum ABC, in quo dentur duo latera AB & AC, cum angulo ad A ab iis comprehenso. Sitque AB part. 50, & ejus sinus rectus AI 7660445; AC partium 60, & ejus sinus rectus FM 8660254; A angulus ab iis comprehensus part. 30, & sinus ejus versus 1339746, deniq; sinus versus differentiae datorum laterum (nempe part. 10) sit 151922. Propositum est ex his invenire tertium latus BC, dato angulo A oppositum. Est igitur per præsens porisma,

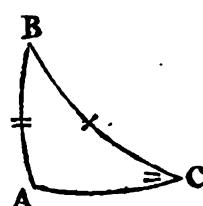
ut	AH	ad FM,	ita AI	ad FN
Item ut	DH	ad FN,	ita DP	ad NO vel LK.
	10000000	8660254	7660445	6634139.
	10000000	6634139.	1339746	888806, differ-

B A S I S

Ex latere & angulo adjacentem, per quintum porisma 13 bujus.

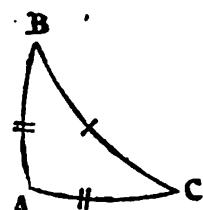


I	II	III	IV	V
ut radius	ad secant. anguli,	ita tang. lateris	ad tangent. basi.	
ut radius	ad sin. compl. ang.	ita tang. compl. lat.	ad tang. comp. basi.	
ut sec. anguli	ad radium	ita tang. compl. lat.	ad tang. comp. basi.	
ut sin. compl. ang.	ad radium	ita tang. lateris	ad tangent. basi.	



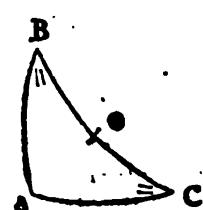
Ex latere & angulo opposito; si constiterit quadrantem major sit, an minor. per tertium porisma 12 bujus.

I	II	III	IV	V
ut radius	ad sec. compl. ang.	ita sinus lateris	ad finum basi.	
ut radius	ad finum anguli	ita sec. compl. lat.	ad secant. comp. basi.	
ut sec. compl. ang.	ad radium	ita sec. compl. lat.	ad secant. comp. basi.	
ut finus anguli	ad radium	ita finus lateris	ad finum basi	



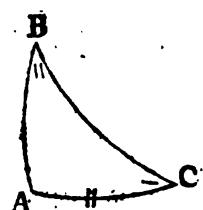
Ex utroque latere, per quintum porisma 12 bujus.

I	II	III	IV	V
ut radius	ad sec. later unius,	ita sec. lat. alter.	ad sec. basi	
ut radius	ad fin. cōp. lat. unius.	ita fin. cōp. lat. ale.	ad finū com. basi	
ut sec. lateris unius,	ad radium,	ita fin. cōp. lat. ale.	ad finū comp. basi	
ut fin. cōp. lat. unius	ad radium,	ita secans lat. alt.	ad secantem basi	

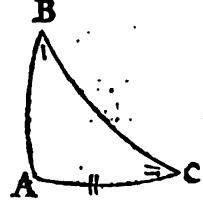


Ex utroque angulo obliquo, per octavum porisma 13 bujus.

I	II	III	IV	V
ut radius	ad tang. ang. unius,	ita tang. ang. alter.	ad secantem basi	
ut radius	ad tan. cōp. ang. uni.	ita tāg. cōp. ang. ale.	ad finū comp. basi	
ut tang. ang. unius	ad radium,	ita tāg. cōp. ang. ale.	ad finū comp. basi	
ut tang. cōp. ang. uni.	ad radium;	ita tang. ang. alter.	ad secant. basi	

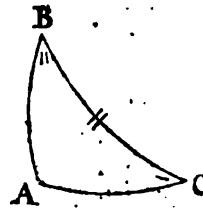
A N G U L U S
Ex latere & dato angulo opposito, si species quæsiti anguli nota sit; per 7 porisma 12 bujus.

I	II	III	IV	V
ut radius	ad secantem lateris	ita fin. cōp. ang. dati	ad finum reliqui	
ut radius	ad finum compl. lat.	ita secans ang. dati	ad secant. cōp. reliq.	
ut secans lat.	ad radium,	ita secans ang. dati	ad secant. comp. rel.	
ut fin. comp. lat.	ad radium,	ita fin. cōp. ang. dati	ad finum reliqui	



Ex latere & dato angulo adjacentem, per sextum porisma 12 bujus.

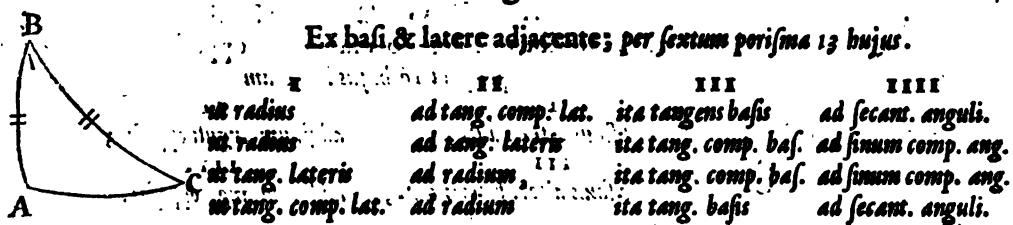
I	II	III	IV	V
ut radius	ad secantem lateris,	ita secans comp. ang.	ad sec. ang. reliqui	
ut radius	ad fin. compl. lateris,	ita finus anguli dati	ad fin. cōp. ang. reliq.	
ut secans lateris	ad radium,	ita finus ang. dati	ad fin. cōp. ang. reliq.	
ut finus compl. lat.	ad radium,	ita sec. cōp. ang. dati	ad secant. ang. reliq.	



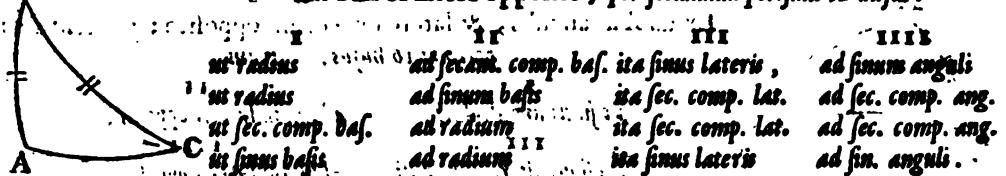
Ex basi & angulo dato, per 7 porisma 13 bujus.

I	II	III	IV	V
ut radius	ad secantem basi,	ita tāg. cōp. ang. dati	ad tang. ang. reliq.	
ut radius	ad fin. compl. basi,	ita tang. anguli dati	ad tang. compl. rel.	
ut secans basi	ad radium,	ita tang. anguli dati	ad tang. compl. rel.	
ut fin. comp. basi.	ad radium,	ita tāg. cōp. ang. dati	ad tang. ang. reliq.	

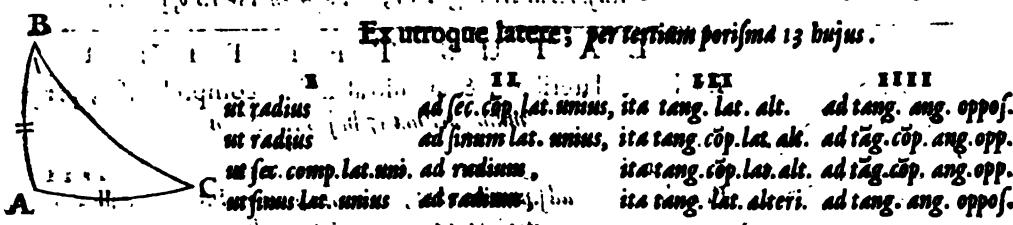
Ex



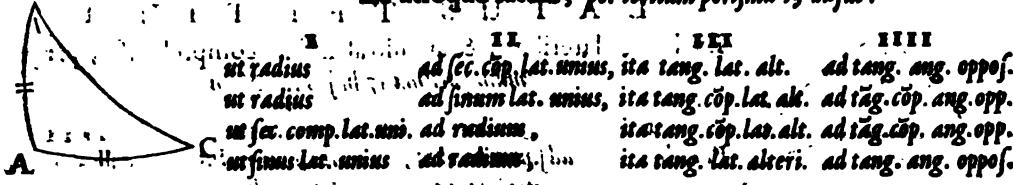
Ex basi & latere adjacentem; per sextum porisma 13 bujus.



Ex basi & latere opposito; per secundum porisma 12 bujus.



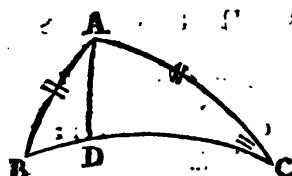
Ex utroque latere; per tertium porisma 13 bujus.



In Obliquangulo Triangulo inveniuntur

L A T U S & A N G U L I D U O.

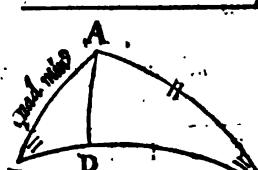
Ex duobus lateribus, & angulo uni eorum opposito; insuper dato specie anguli alteri dato latere oppositi: per primum porisma 15 bujus.



Arches enim perpendicularis demissus ab angulari puncto datorum laterum in tertium latum; continuatum si necesse sit, secat obliquangulum triangulum datum in duo triangula rectangula: ex quorum calculo quaestia dicuntur.

A N G U L U S & L A T E R A D U O.

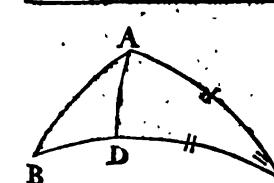
Ex duobus angulis & latere uni eorum opposito; si constet utrum tertium latum quadrante majus sit, an minus:
per secundum porisma 15 bujus.



Perpendicularis figurans arcus a termino lateris dati in Latus utriusque angulo dato adjacens (continuum si operatur) descriptus, partitur obliquangulum triangulum datum in duo Triangula rectangula; ex quorum datis postulata immotescunt.

L A T U S & A N G U L I D U O

Ex duobus lateribus, & angulo ab iis comprehenso; per tertium porisma 15 bujus.

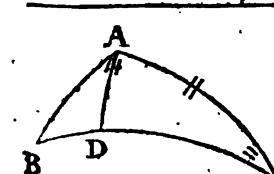


Arches enim perpendicularis, a termino lateris unius dati emissa in alterum latum datum (productum si necesse sit) obliquangulum triangulum in duo triangula rectangula dividit; ex quorum calculo ignota colliguntur.

A N G U L U S & L A T E R A D U O

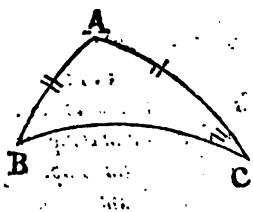
Ex duobus angulis & latere utriusque angulo adjacentem, per quartum porisma 15 bujus.

Nam arcus perpendicularis ab angulo alteriore in oppositum Latus (continuum si necesse sit) egrediens, obliquangulum Triangulum in duo Triangula rectangula secat, ex quorum calculo postulata dicuntur.



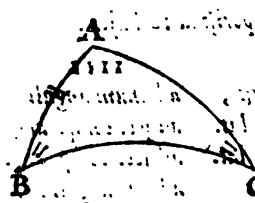
Geometriæ Triangulorum Liber IIII.

A N G U L U S



Ex duobus lateribus & angulo uni eorum opposito; per primum porisma 16 bujus. Nam

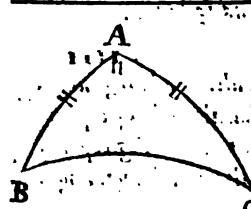
VI finis lateris dati, ad finem anguli oppositi;
ita finis alter. lateri dati, ad finem ang. oppositi.



L A T T U S T U S

Ex duobus angulis, & latere uni eorum opposito; per secundum porisma 16 bujus. Nam

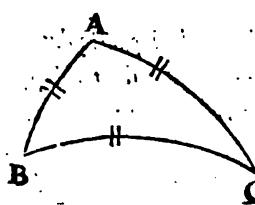
VI finis anguli dati, ad finem lateris oppositi,
ita finis alter. ang. dati, ad finem lateris oppositi.



L A T T U S T E R T I U M

Ex duobus lateribus, & angulo ab iisdem comprehenso, per primum porisma 17 bujus. Nam

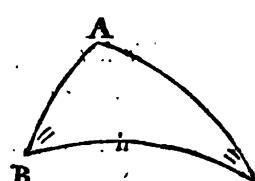
VI radius ad finem rectam, ita finis rectus lat. unius ad quartum.
VI radius ad quartum ita finis versus anguli dati ad differentiam finium versorum tertii lateri, & reliquum laterum differentia. Hac vero differentia ad finem versus differentia laterum addita, componit finem versus lateris quesiti.



A N G U L U S Q U I V I S

Ex tribus lateribus; per secundum porisma 12 bujus. Nam

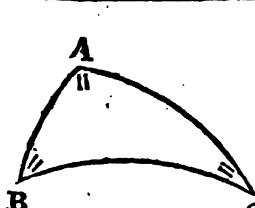
VI radius ad finem rectum ita finis rectus lat. unius, ad quartum.
VI radius ad quartum ita finis versus ang. alterius ad quartum.
VI radius ad radium, ita differentia finium vers. tertii lat. & reliq. laterum differentia. ad finem versus ang. quesiti.



A N G U L U S T E R T I U S.

Ex duobus angulis, & latere utrique angulo adjacente; per tertium porisma 17 bujus. Nam

VI radius ad fin. rectum ang. unius, ita fin. rectum ang. alterius ad quartum
VI radius ad quartum ita finis versus lateris dati ad differentiam finium versorum quesiti anguli, & differentia anguli unius dati, & alterius ad semicirculum residui. Differentia igitur hec addita ad finem versus anguli unius dati, & reliqui ad semicirculum complementi, componit finem versus anguli quesiti.

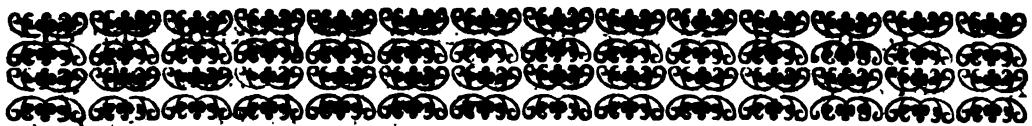


L A T T U S Q U O D V I S.

Ex tribus angulis; per quartum porisma 17 bujus. Nam

VI radius ad fin. rectum ang. unius, ita fin. rectum ang. alter. ad quartum
VI radius ad radii, ita differentia finium vers. tertii ad finem versus angul. & differentia ang. unius, lateris quesiti. & alter. ad semicirculum residui.

Mixtu tripli dico.



PHILIPPI LANSBERGII
CYCLOMETRIÆ NOVÆ
LIBRI DUO.

Illustrissimo Principi ac Domino D. Mauricio Princi-
pi-Aultaico, Comiti Nassovio, &c. Gubernatori Belgii con-
federati, & ~~Academy~~, &c.

E T

*Illustribus ac Potentibus Zelandie Ord. Dominis ac Mecenatibus suis fibi
plurimum venerandis.*

 **I**R C U L I geodesia, quam magnus Archimedes ~~invenit~~ appellat, propter utilitatem quā societati hominum atque communitati adfert insignem, jam multis ab hinc seculis ubivis gentium exulta est. Et primū ante annos bis mille & sexingentos in Palaestina, sub magni Solomonis imperio. Tunc enim inter cætera templi ornamenta intestina, constructum fuit mare æneum circumquaq; rotundum, factaque ipsius dimensione, deprehensum, quod decem cubiti essent à labii parte unâ ad alteram, & quod filum triginta cubitorum idem cingeret circumquaque. Erat itaque tum temporis Cyclometria quadam in usu, rudis scz. illa, quæ diametri & peripheriaz rationem ponebat triplam, hoc est, ut X ad XXX.

At septingentis annis post Solomonem, circa Platonis tempora, accurratior quædam circuli dimensio in Græcia caput efferre cepit, quando magni Viri Bryso, Antipho, Hippocrates Chius, Cyclometrica sua inventa, dabant in publicum, laudemq; Cyclometricaz inventaz singuli affectabant. Brysonem enim excipiebat Antipho, Antiphontem Hippocrates, atque hunc deinceps alii, manente tamen Cyclometricaz laude penes Hippocratem. Nam ut testis est Aristoteles, Brysonis ~~ingeniosus~~ erat ignorans, ~~indolens~~; Antiphontas reprehensione Geometrica indignus. Hippocratis contra qui fiebat per ~~ingeniosus~~ vere erat Geometricus: quo tamen posteritas minimè fuit contenta, quod non tam circuli esset, quam duorum circuli ~~ingeniosus~~.

M

Hos

Hos omnes, qui ~~erant~~ erant, ducentorum proxime annorum intervallo sequutus est Archimedes Syracusanus, vir mirae sagacitatis, qui ~~erat~~ Antiphontis mutuatus duo demonstravit; unum, cuiuslibet circuli circumferentiam suæ diametri esse triplam, & adhuc excedere minori quidem parte quam septimam diametri, magis autem quam decem septuagismis primis. Alterum, Omnes circulum æqualem esse triangulo rectangulo, cuius unum latus circa rectum æquatur radio circuli, alterum perimoto. Quæ elementa et si posteritas agnoverit, ab Archimedea esse demonstrata, non aquievit tamen in ipsius Cyclometria, quam tertio clemento proposuit, quod à veritate non nihil distaret. Testes sunt Apollonius Pergæus magnus Geometra, Ptolemaeus Alexandrinus Astronomæ Princeps, Philo Gadarenus, atque alii quorum Eutocius Ascalonita meminit, qui omnes summam ope ac studio conati sunt Cyclometricum negotium ~~in~~ ~~ad~~ ~~en~~ ~~en~~ ~~en~~. In quam quoque cogitationem & curam incubuerunt Dinostratus, Nicomedes, & quotquot deinceps Mathematici præstantes sequuti sunt ad nostra usque tempora.

Porro cum nobis valde doleret, nobilissimam Geometriæ partem, in qua tot seculis totque à Geometris sudatum esset, in splendorem suum non modò nondum esse redditam, sed ~~quod~~ multorum magis foedatam, ingensq; restituendi eam cupiditas animum nostrum incesseret; admovimus & nos, ~~in~~, operi manum, observataq; circuli naturam, ac signis quæ illi insunt, tandem post aliquot annorum vigilias, & pertinacem cogitandi assiduitatem, Cyclometriam de integro extruximus, novam quidem, sed quæ cum vetere de palmam certare audeat. Hanc Flacci consilio nonum pressimus in annum: verum quia ultra jam premi negat, bonorei literariæ manumittimus. Stat enim in carceribus ~~in~~ ~~in~~ ~~in~~ ~~in~~.

Ut autem in manus hominum veniret gravior, volui eam Illustrissimæ T. C. ac D D. V. Illustribus & Potentibus inscribere, Archimedem sequutus, qui partem operum suorum optimam dicavit suo Dositheo. Et si enim Mathematica per se Mathematicis Viris sint accepta, tamen quia persæpè incidunt in manus ~~etiam~~, utile est laudatorum Virorum nominibus esse insignita, ut qui ab illis abhorrent maximè; eorum saltem exemplo ad ea invitentur.

In primis decet ut qui Mathematica tractant publicè, bonorum ac sapientum Principum mentionem faciant. Nam quia boni & sapientes Principes hæc studia præcipue juvant, consentaneum est

ut ii quibus opem ferunt, vicissim virtutem ac liberalitatem eorum agnoscant, quantumque fieri potest, grato animo prædicent. Quam obrem cùm & vos esse Mathematicarum artium fautores constet, & te, Princeps Illusterrime, inter Mathematicos nostri seculi primum; æquum esse putavi ut & ego sapientiæ ac virtutum Vestrum ornamenta publico testimonio cōprobarem: præsertim cum totos triginta annos benevolentia Vestrae aurâ fuerim afflatus, jamque in hoc meo senio, summo Vestro favore ac magnificentiâ, ocio fruar literario. Agnosco enim & me hoc nomine Vobis debere plurimum, & illos quoque qui deinceps ocii nostri fructum percipient.

Oro itaque te Illusterrime Princeps, Vosque Ordd. Illustres & Potentes, quām possum reverenter, Cyclometriam ut hanc nostram, in speciem quidem exiguam, sed materia & labore maximum, patiamini sub Illusterrimis V. Nominibus venire in lucem; carnque extare ut publicum observantiæ ac gratitudinis meæ erga Vos monumentum. Hoc enim animo eandem Vobis do, dico, consecro, cupioque ut quæ laus inde expectanda est, Vobis cedat; Quibus jam pridem me totum devovi; Quibusque jam studia mea sub misse commendo. Vale Illusterrime Princeps, & Vos Ordd. Illustres ac Potentes. Middelburgi Zelandiæ, pridie Idus Januar.

c l o i c x v i .

*Illustriſſa T. Celsitudini, ac D D. U. Illustribus &
Potentibus*

Addictissimus

P. L A N S B E R G I U S.

Ex literis Clarissimi, Doctissimique viri Willebrordi Snellii, R. F. ad Philippum Lansbergium.



U A M verò novam quadraturam non leviter admiror, qui etiam in vastissimis illis numeris tam prope verum collimaris; unde facile conjecturam capio, eam vero assidere. Quod tamen Cl. Ludolphum in ratione diametri & peripherie constituenda hallucinatum censes, haud possum tecum sentire: neque enim est ratio legittima quam ponis 2000000000000000000000000 ad 62831853071795864769128. Sed ista 2000000000000000000000000 ad 62831853071795864769252867 & amplius. Habes numeros tuos maiores. Ceterum verissime illud notasti omnes Veterum quadraturas in numeris maximè expediri. Nam volutas Sisyphus aliquis per puncta & pinnas rotabit, volvetque. Apage mihi istas, quia à π sunt alienissimæ. Quare quod ais ex tuo invento rectam circulo quadrando proximè æqualem ita describi, ut facilitas cum certitudine contendat, eo ipso sine dubio mirificè hominum studia excitabis. Imò quis tibi non plaudet, cùm videbit unum aliquem hac ètate inventum, qui non desperarit secula proficere semper? qui non dubitarit cum summis Mathematicis de prima summâ que Mathesios laude decertare. Archimedem redivivum, quadratores novum te amplectetur hæc ètas, agnoscent posteri, qui primus post Veteres novo epichiremate id opus tentaris & perfeceris; & propterea grato animo tuam memoriam recolent, cum tanto & tam amplio patrimonio mathematicam rempublicam à te auëtam & locupletatam cogitant. Imò nos beabis, cum quadratores novos, qui è nido evolant crocitatores odiosi & molesti, nobis amplius inolectos esse non sines. Est enim quoddam hominum genus insolita temeritate, & impudenti audacia, qui simul ac oculos Barbarico cœno infossos pauxillum mathematico sole clariores habent, statim in circuli quadraturam tanquam materiam suis viribus aptam involant, atque ea quæ ex antiquo ignorantie cœno haulere, nobis tanquam maxima rata sententias obrrudore conantur. Hinc tanta ~~videtur payasim~~, omnibus èvis copia, neque enim hoc hominum genus demum ~~videtur payasim~~ vobis antegressus norabit, vetus hoc malum est, & Veteribus quoque adgnitum. Atque ideo tanto impensiùs te togo Clarissime L A N S B E R G I, ne illa diutius premias, neve anteactas tuas virgilias sinas interire, quod futurum fuisse scribis, nisi aliquando à nobis excitatus, & in antiquam palestram penè reductus es. Neque enim decet te Aspendium citharistam imitari, quem omnia intus canere dicebant, ut tu quoque tibi soli sapias; sed multo magis ut publicè prodis, & Belgici nominis claritatem nunc ad exterios, olim verò ad posteros propages. Vale Vir Clarissime, & affectum quo nos hactenus complexus es deinceps porro continuo. Lugduni Batavorum x. Octobr. c I o I o c v i i .

Tuus, tibiisque addicfimus

WILLEBORD. SNELLIUS, R. F.

Lectori benevolo S.

H Abes, Lector benevole, Cyclometrica nostra, jamdiu à nobis efflagitata; quæ si grata tibi esse cognovero, dabo operam ut Astronomica nostra, saltem pars eorum prima de motu diurno, annuo, mensu, mox ad te perveniat. Vale bone Lector, & studiis nostris fave.

C Y C L O -

C Y C L O M E T R I A E

L I B E R I.

De dimensione circuli ambitus.

1. *Cyclometria est pars Geometrie que circulum bene metiri doceat.*

 UOD magnus Archimedes ~~etiam nigrum~~ appellat, nos una voce Cyclometriam dicimus. Pars est Geometriæ nobilissima in qua se exercuerunt præstantissimi Geometriæ, prisco quidem seculo, Bryso, Antiphœ, Hippocrates Chœn, Dinostratus, Euclides, Archimedes Syracusanus, Appollonius Perganus, Ptolemæus, Nicomedes, Pappus Alexandrinus, Sporus Nicenus, Philo Gadarenus, Eutocius Ascalonita, Boëtius, Campanus, & alii: nostro verò & Proavorum ævo; Nicolaus Cusanus Cardinale, Ioannes Regiomontanus, Orontius Delphinalis, Jacobus Peletarius, multique post illos, quorum nomina referre non est opus. Cæterum et si inter omnes quos dixi magnus Archimedes Cyclometricum negotium maximè promoverit, haud satis tamén elaboratam fuisse ipsius Cyclometriam, quotquot eum celebres Geometriæ sequuti sunt, ad unum omnes judicarunt. Hinc factum est, quod qui post ipsius tempora in genio & Mathesis scientia insignes fuerunt, vites omnes intenderint, ut Cyclometriam Archimedæ ~~invenient~~ darent. Ego verò et si minimus sim omnium quos dixi, audeo tamen in Cyclometricam arenam descendere, & polliceri, Cyclometriam quam nunc profero in lucem, Veritati & Geometriæ principiis magis esse consentaneam, quam, Geometrarum qui nos præcesserunt. Quod tamen non arroganter, sed pro rei veritate ingenuè dictum esse, in sequentibus, Deo volente, satis superq[ue] evincam.

2. *In circulo ad bene metiendum duo proponuntur, circuli ambitus, & area.*

Tria in circulo considerantur, centrum, peripheria, superficies, vel area. Centrum verò quia puncti locum obtinet, magnitudinis est expers. Peripheria verò & superficies, quia magnitudines sunt, sub mensuram cadunt; utraque igitur in circulo ad bene metiendum proponitur.

P O R I S M A.

Iaque Cyclometria duabus partibus absolvitur, ambitus circuli dimensione & area.

Porismatis consequentia manifesta est. Quia enim in circulo duo tantum ad bene metiendum proponuntur, ambitus circuli & area, necesse est Cyclometriam duabus tantum partibus absolvi, Dimensione ambitus circuli & area. Quare de illis sigillatum agendum est.

3. *Ambitum circuli dimetiri, est non modo rectam describere cuiusvis circuli propositi peripherie aqualem, & cunctaque recte date aqualem circuli peripheriam; sed rationem quoque explicare quam inter se habeant peripheria cuiusvis circuli dati & diameter.*

Cyclometriae Liber I.

Ambitus circuli dimensio vel Geometricè instituitur, vel Arithmeticè. Si Geometricè, oportet rectam lineam describere circuli propositi peripheriaz à qualem, vel rectæ dataz à qualem circuli peripheriam. Sin Arithmeticè, definienda est ratio, quam inter se habent peripheria data & diameter. Archimedes utrumque facere conatus est. Nam 18. m. i. 2. rectam lineam ducere instituit circuli dati peripheriaz à qualem. Secunda verò propositione 20. m. i. 2. cuiuslibet circuli peripheriaz rationem ad diametrum definire tentat. Quare & nobis utrumque est præstandum.

4. Si peripheria sinus aut tangens, ad dimidie peripheria sinum aut tangentem fuerit, ut peripheria ad peripheriam dimidiā, peripheria, sinus, tangens, inter se àquales erint.

Sinus & tangentes peripheriis àquales voco, non qui absolutè àquales sunt, sed qui àequalitatē habent, saltem in dato círculo, vel circulis dato círculo minoribus. Absolutè enim nullus sinus aut tangens peripheriaz suis est àqualis. Nam quia omnis inscripta minor est sua peripheria, & circumscripta omnis major, oportet etiam semisses inscriptarum, id est sinus peripheriis suis esse minores; & circumscriptarum semisses, hoc est tangentes iisdem majorē. Hippotetice verò sinus & tangens arcui suo àqualis est, quando eorum discrimen nullum ostendi potest in dato círculo. Nam ut acutissimus Geometrarum nostri seculi Nicolaus Copernicus anaœavit lib. Revolut. i. cap. 12. problemate ultimo, inscriptæ, adeoque & sinus & tangentes, per continuam biflectionem peripheriarum tendunt ad àequalitatem, tandemq; ad extremitatum circuli contactum àquales fiunt acsi una linea essent.

Dico igitur peripheriam, sinus, tangentem esse inter se àquales, si peripheriaz sinus vel tangens sit ad sinum vel tangentem peripheriaz dimidiz, ut peripheria ad peripheriam dimidiā. Nam si inæquales essent, etiam per demonstrata Ptolemaei libro m. 1. cap. 9. essent disproportionales. Atqui ex hypothesi proportionales sunt, ergo etiam inæquales. Nam proportionem hīc semper sequitur àqualitas, & inæqualitas disproportionem. Illustrē, exemplum subministrat Canon Sinuum & Tangentium in peripheriis grad. o. 10, & grad. o. '5. Illius enim & sinum & tangentem eundem exhibet particul. 29088, hujus verò particul. 14544, in mensura radii 10000000.

Sunt autem hi sinus & tangentes peripheriis suis primum proportionales. Nam peripheria grad. o 10, se habet ad peripheriam grad. o. '5, ut sinus vel tangens 29088, ad sinum, vel tangentem 14544.

Secundū idem sinus tangentibus suis àquales sunt. Nam peripheriaz grad. o. 10, idem est sinus & tangens particul. 29088; idemque est sinus & tangens peripheriaz grad. o. '5. particul. 14544.

Tertiò ipsis Sinus & Tangentes peripheriis suis àquales sunt. Quia enim sinus tangentibus suis àquales sunt, oportet etiam peripheriis suis àquales esse, quæ tangentibus absolutè sunt minores. Item quia tangentes sinibus àquales sunt, necesse est peripheriis suis quoque àquales esse, quæ sinibus suis absolutè sunt majorē. Itaque peripheria, sinus, tangens, inter se àquales sunt, cum peripheriaz sinus vel tangens est ad sinum vel tangentem, peripheriaz dimidiz, ut peripheria ad peripheriam dimidiā. Quod erat demonstrandum.

5. Si dati circuli quadrans per biflectionem in quatuor partes àquales dividatur, radiusque erectus in partes àquales rotidem; & à punto divisionis radii ultimo, per divisionis quadrantis punctum ultimum recta ducatur in ultimi arcus tangentem; abscedet hec ex dicta tangentem arcui quadrantis ultimo àqualem.

Hoc Theorema totius Cyclometriae fundamentum continet. Quare perspicuè explicari, accuratèque demonstrari debet.

Datiū ictū círculum appello, cujus radius in certa mensura data est, puta 10, 100, 1000, 10000, 100000, 1000000, 10000000, vel quacunque alia.

Cyclometric Liber I.

90

Ultimum arcus Quadrantis dico, qui peripheria Quadrantis, vel semicirculii, videlicet libet, biseccit, est ultimus.

Denique rectam ultimo Quadrantis arcui aequalem dico, non quae talis est in ordinarii circulo, sed sicutem in dato.

Esto jam in adjuncto schemate Quadrans circuli ABCD, cuius peripheria BCD & radius erectus AB bisecentur, ille in C, hic in E; ducaturque ab E bisectionis radii puncto, per C bisectionis Quadrantis punctum recta ECF in DG tangentem ultimi arcus DC. Dico ECFC secantem absindere ex tangente DG, tangentem DF, aequalem ultimo quadrantis arcui DC.

Demonstratio perspicua erit si semiradius AE biseccetur in H, & DC semiquadrans in I, & ex punto H per punctum I ducatur recta HIK in tangentem DG. Hac enim quia tangentem DF abscissam bifecabit in K, erit DF ad DK, ut peripheria DC ad peripheriam dimidiad DC, adeoque per præmissum elementum, DF tangens abscissa, aequalis erit ultimo arcui DC, & illius semissis DK hujus semissi DL. Quorum veritas cum in numeris sit maximè conspicua, subiicio sequentem calculum.

Sit AB radius particul. 10, vel 100 (liber enim metiri circulum, omnium qui dari possunt minimum) eritque AE semiradius particul. 50, & CD semiquadrans grad. 45, quorum BCD totus quadrans est 90. Demittatur quoque perpendicularis CL ex C termino arcus DC in radium AD; erit hac sinus rectus arcus DC particul. 70, qualium AD radius est 100, & AL vel EN sinus complementi itidem particul. 70. Præterea ex E in tangentem DF ducatur recta EM parallela AD, quæ CL secet in N; tandemque

$$\text{auferatur ex CL } 70.$$

$$LN \text{ id est AE } 50.$$

$$\text{eritque residua NC } 20.$$

Quoniam vero triangula EMF & ENC sunt similia, propter rectos angulos ad M & N, communem ad E, per 4^m Sext. Euclidis est,

$$\begin{array}{rcl} \text{Ut } EN 70, \text{ ad } NC 20, \text{ ita } EM \text{ id est } AD 100 \text{ ad } MF & 28\frac{6}{10} \text{ proxime. cui} \\ & \text{si addas } DM & 50 \\ & \hline \text{Componitur } DF & 78\frac{6}{10} \end{array}$$

DF itaque est particul. 78 $\frac{6}{10}$ qualium AB radius est 100.

Definienda deinceps est quantitas DK in eadem mensura radii. Quia igitur AE est particul. 50, semissis ejus AE est particul. 25. Item quoniam arcus CD est grad. 45, ejus dimidius DI est grad. 22 $\frac{1}{2}$, ejusque sinus rectus IO particul. 38 $\frac{1}{2}$ in mensura radii 100, & complementi sinus AO id est HQ 92 $\frac{1}{2}$.

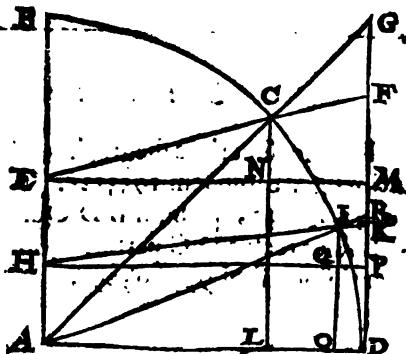
$$\begin{array}{rcl} \text{Subducatur vero & hic ex IO } 38\frac{1}{2} \\ QO \text{ id est AH } 25 \\ \hline \text{reliqua erit QI } 13\frac{1}{2} \end{array}$$

Itaque per 4 Sexti Euclidis ut supra

$$\begin{array}{rcl} \text{Ut HQ } 92\frac{1}{2} \text{ ad QI } 13\frac{1}{2}, \text{ ita HP } 100 \text{ ad PK } 14\frac{1}{2}. \text{ Cui si} \\ \text{addas DP } 25 \\ \hline \text{Componitur DK } 39\frac{1}{2} \end{array}$$

Hinc autem manifestum est rectam HIK biseccare DF in K; Est enim

Ut DF 78 $\frac{6}{10}$ ad DK 39 $\frac{1}{2}$, ita DC arcus grad. 45 ad DI arcus grad. 22 $\frac{1}{2}$. Quare per præcedens elementum recta DF aequalis est arcui DC, & recta DK arcui DI, in quadrante



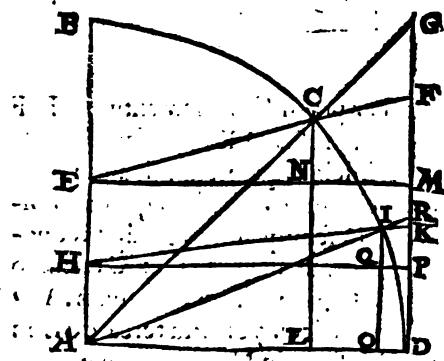
drante A B D, cuius radius datas est particul. solummodo 100. Quod erat demonstrandum.

Et hæc quidem est Theorematis nostri demonstratio, adeo firma, ut nulla ejus particula cum ratione possit convelli. Ut tamen ipsius veritas manifestior reddatur, subiicio plura exempla, ex quibus apparebit, quod in uno ostensum fuit, verum esse in omnibus.

Secundum exemplum ex magno Canone Rhetici.

Retenta superiore diaphaga; esto radius A B particul. 1000000000, & A E pars ejus ultima $\frac{1}{11}$, earundem 1953125. Sit etiam arcus D C $\frac{1}{11}$, pars Quadrantis B C D, grad. o. 10. 32. 48. 45, qualium B D Quadrans est 96. Detur quoque ex magno Canone Rhetici C L sinus rectus D C particul. 3067956, quarum radius A B est 1000000000, & comple-

menti sinus E N 999995293.



Ablato primum ex C L	3067956
Ultima parte radii L N	1953125
relinquitur N C	1114831

Unde per 4 Sexti Euclidis

Ut E N 999995293 ad N C	1114831,
ita E M 1000000000 ad M F	1114836.
cui si addas ult. part. rad. D M	1953125

Erit D F 3067956

Bisera jam A E 1953125, & arcum D C grad. o. 10. 32. 48. 45, erit A H 976562 $\frac{1}{2}$ & D I grad. o 5 16 24 22 $\frac{1}{2}$, ejusque rectus sinus I O 1533980, & complementi H Q 99999882 $\frac{1}{2}$.

Porro & hic ex I O	1533980
subducatur A H	976562 $\frac{1}{2}$
reliqua erit Q I	557417 $\frac{1}{2}$

Quamobrem per 4^m Sexti Euclidis

Ut H Q 99999882 $\frac{1}{2}$ ad Q J	557417 $\frac{1}{2}$
ita H P 1000000000, ad P K	557418. Cui
si addas D P	976562 $\frac{1}{2}$

Erit D K 1533980 $\frac{1}{2}$.

Unde iterum manifestum est rectam H I K bisecare D F in K; est enim

Ut D F 3067956 ad D K 1533980, ita arcus D C ad arcum D L

Quamobrem per præcedens elementum, D F est æqualis D C, & D K est æqualis D I. Quod erat ostendendum.

Cæterum ne quis existimat aliter se habere in radiis majoribus, addam unum atque alterum exemplum radiorum majorum. Utque ~~in~~ expeditior sit, sum in sequentibus exemplis, tum in reliquis omnibus, præmitto tres Canones, quorum magnus est usus in Cyclometrico calculo.

Primus Canon continet continuas ~~longinas~~ radii vastissimi, viz. particularum.

100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 0.

Exhibit autem hic Canon in ima parte, radii particularis; in sinistro margine bisectionis numerum, hoc est, quoties radius bisectus fit; in area communi, particularis ultimæ partis radii.

Cyclometria Liber).

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Canon continuo duum radii qui ponitur particulariter.

100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 0.

1	5
2	25
3	125
4	625
5	3125
6	1562, 5
7	781, 25
8	390, 625
9	195, 3125
10	97, 65625
11	48, 82812, 5
12	24, 41406, 25
13	12, 20703, 125
14	6, 10351, 3625
15	3, 09175, 98125,
16	1, 52587, 89062, 5
17	76293, 94531, 25
18	38146, 97265, 625
19	19073, 48632, 8125
20	9536, 74316, 40625,
21	4768, 37158, 203125, 5
22	2584, 18579, 10156, 25
23	1192, 69289, 55078, 1250
24	596, 04644, 77539, 0625
25	298, 02322, 38769, 53125,
26	149, 01164, 19384, 76562, 5
27	74, 50580, 59692, 38281, 25
28	37, 25190, 29846, 19140, 625
29	18, 62645, 24923, 09570, 3125
30	9, 31322, 57461, 54785, 15625,
31	4, 65661, 28730, 77392, 57812, 5
32	2, 32830, 64365, 38696, 28906, 25
33	1, 16415, 32182, 69348, 14453, 125
34	58207, 66091, 34674, 07226, 3625
35	29103, 83045, 67337, 03613, 28125,
36	14451, 91522, 83668, 51806, 64062, 5
37	7275, 95761, 41834, 25903, 82031, 25
38	3637, 97380, 70917, 12951, 66915, 625
39	1818, 98940, 35458, 56475, 83007, 8125
40	909, 49470, 17729, 28237, 91503, 90625,
41	454, 74735, 08864, 64118, 95751, 95312, 5
42	227, 37367, 54432, 32059, 47875, 97656, 25
43	113, 68683, 77216, 16029, 73937, 98828, 125
44	56, 84341, 88608, 08014, 86968, 99414, 0625
45	28, 42170, 94304, 04007, 43484, 49707, 03125
46	14, 21095, 47152, 02003, 71742, 24853, 515625
47	1, 00005, 00000, 00000, 00000, 00000, 00000, 00000, 0.

N

In

Cycloometriae Liber I.

In exemplo, si detur radius particul. 100000, 00000, 00000, 00000, 00000, 000, ultima pars radii est 37252, 90298, 46191, 40625. numerentur enim in ima parte Canonis circuli 28, & à postremo circulo ascendatur directè ad numerum ultimum, erit hic numerus ultimus numerus postremæ partis radii, viz. 37252, 90298, 46191, 40625. Numerus autem 28 in sinistro margine, ultimæ parti radii respondens, indicat quoties radius datus bisectus sit, nimirum vicesies & octies.

Item si detur radius partic. 100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 0, ultima pars radii est, 14210, 85471, 52020, 03717, 42248, 53515, 625. Nam si & hic in ima parte Canonis, circuli 46 numerentur ab unitate radii, & ab ultimo circulo sursum ascendatur directè ad ultimum numerum, erit hic numerus ultimæ partis radii, numerusque 46 in sinistro Canonis margine, docet quoties radius sit biseptus, viz. quadragesies & sexies. Et hic quidem est primus Canon; sequitur alter.

Canon continua Æquanim peripherie Quadrantis.

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096
13	8192
14	16384
15	32768
16	65536
17	131072
18	262144
19	524288
20	1048576
21	2097152
22	4194304
23	8388608
24	16777216
25	33554432
26	67108864
27	134217728
28	268435456
29	536870912
30	1073741824
31	2147483648
32	4294967296
33	8589934592
34	17179869184
35	34359738368

Hic Canon exhibet continuam bisectionem peripherie Quadrantis, à prima bisectione usque ad quadragesimam sextam. Licet autem ex hoc Canon, vel uno intuitu cognoscere quota pars Quadrantis sit ultimus arcus ex continua bisectione factus. Numerus enim in sinistro margine ostendit quoties datus quadrans sit biseptus; et qui in area se offert, docet quota pars Quadrantis sit arcus à bisectione ultima factus.

In exemplo detur peripherie Quadrantis, cuius radius sit particul. 100000, 00000, 00000, 00000, 00000, 000, Ex superiori Canone constat (ut etiam ex numero circulorum radii) ultimam partem radii fieri ex bisectione ipsius radii, vicesies & octies continuata. Aequi & ultimus Quadrantis dati arcus sit ex bisectione Quadrantis octies continuata. Quare ut numerus 28 in praemissio Canone præbet partem ultimam radii; ita in praesenti Canone ultimum arcum Quadrantis nullum, 6438456. Quasim itaque peripherie datu Quadrantis particularum est 268435456. Ultimus quadrantis arcus est una particula.

Eodem modo ultimus arcus peripherie Quadrantis, cuius Radius ponitur particularum 100000, 00000, 00000, 00000, 00000, 000, 00000, 00000, 00000, 00000, 00000, ex praesenti Canone obtainetur, 29367244776. Superior enim Canon (ut & pumerus circulorum radii) docet bisectionem Radii quadragesies & sexies esse continuandam. At ut numerus 46 in praemissio Canone dat ultimam partem Radii; ita in praesenti Canone dat ultimum arcum Quadrantis, viz. 703684417766. Quasi itaque Quadrans circuli datu est particularum 703684417766, arcus quadrantis ultimus est particula una.

Tertius Canon continet subtentas complementorum arcium ad semicirculum, qui ex continua bisectione Quadrantis oriuntur, idque in mensura Radii vastissimi particul. 100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000. Quem Canon erga summa industria, atque indefesso labore supputavit logaritmi negoti regni Princeps Ludolphus à Colle, eundemq; abhinc octennium nobiscum perhumaniter communicavit. Licet autem subtentas ista docira calculi molestiam supputare peripheriarum sinus, quæ ex continua

Reliquum precedentius Canonis.

36	68719476736
37	137438953472
38	274877906944
39	549755813888
40	2099511627776
41	219902325552
42	4398046511104
43	8796093022108
44	1759218604416
45	35184372088832
46	70368744177664

Quadrantis bisectione proveniunt, in mensura radii 100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000. saltem ad bisectionem Quadrantis 46^m. Tot enim subtensas Canon exhibet; neque plures desiderantur, respectu Canonum præmissorum, qui quadragesimam sextam Radii & Quadrantis bisectionem non transeundunt.

Ratio autem subtensarum hæc est. Prima subtensa est peripheria Quadrantis. Secunda differentia; peripheria Quadratis & semicirculi. Tertia differentia; peripheria Quadratis & semicirculi, atque ita deinceps usque ad quadragesimam sextam, quæ est subtensa differentia 70368744177664 peripheria Quadratis & semicirculi.

Primæ autem subtensæ & Diametri differentia latus quadratum est subtensa; peripheria Quadratis; itaque semissis ejus est sinus; Quadrantis. Secundæ subtensæ & diametri differentia latus quadratum est subtensa; peripheria Quadratis, & semissis ejus est sinus; Quadrantis. Tertiæ subtensæ & Diametri differentia latus quadratum est subtensa; peripheria Quadratis, & semissis ejus est sinus; Quadrantis, atque ita deinceps. Unde manifestum est quomodo ex Canone subtensarum cuiusvis arcus ultimi & complementi sui sinus investigandi sint. Primum ex numero circulorum Radii dati, vel ex præmissis Canonibus colligendum est, quota quadrantis bisectionis ultimus quadrantis dati arcum. Deinde cum numero quoto ingredi oportet Subtensarum Canonem, & subtensem sumere quæ quoto numero responderet. Hujus semissis est sinus complementi arcus ultimi dati. Latus vero quadratum differentia subtensæ immediatè præcedentis & Diametri, est sinus rectus ipsius arcus ultimi. Ecce autem Canonem ipsum.

Canon subtensarum arcuum peripherie Quadrantis continuè bisectionis.

P A R T I C U L A E D I A M E T R I.

1	141421356237309504880168872420969807856967187537694807
2	184775906502257351225636637879357657364483325172728497
3	196157056080646089825236447226847807394786746178667219
4	199036945334439377248967390621895984315094973745971412
5	199759091241034478542954320951820138888640722940922373
6	199939763739240844023153129933234439370012216251545928
7	199984940367828908184325298239276644870121293760443566
8	199996235056520228531398087545713543234783450188867018
9	1999990587619152343023160251400239799105975267244375308
10	1999997646903403819858051420343052038096535845779530078
11	1999999411725764438320456435477531354232527798698603203
12	199999852931435702289462961414775713896402311377845532
13	199099996322858587616693830805819429610152102485560705
14	1999999990808214625781943866279212297917788606378905647
15	1999999997702053655125346615589108216801074832388564246
16	1999999999425513413698827944437283552178178915836570505
17	1999999999856378353419550191767700980520960666219031603
18	1999999999964094588354565248295568214759278374992551601
19	1999999999991021647088621768345994648883029202930628122
20	1999999999997755911772154033103505073007698443821045219

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Reliquum precedentis Canonis.

21	19999999999999438977943038429589439168904036019000060424
22	19999999999999859744485759602479457473516421086293467729
23	1999999999999964936121439900312495473459750637897030419
24	19999999999999991234030359475058913312432477910669911536
25	199999999999999997808507589993763527668362340742051587894
26	199999999999999999945212689749840806875856474014516347312
27	19999999999999999986303172437461019702886986555441481278
28	19999999999999999965757931093652548964091925892098639953
29	1999999999999999999914394827734131372227023676352051762
30	1999999999999999999785987069335284304510553556795974925
31	199999999999999999994646767333832107601073491555741355
32	19999999999999999998662419183345802699310795566786232066
33	1999999999999999999966560479583645067252419341315139062
34	199999999999999999991640119895911266813087363429946677
35	199999999999999999997910029973977816703270748863809101
36	199999999999999999999477507493494454175817618966347436
37	1999999999999999999986937687337361354395440475986557
38	1999999999999999999996734421834340338598859852396620
39	19999999999999999999991836054585850846497149946436654
40	199999999999999999999979590136462711624287485567757
41	1999999999999999999999948753411615677906074871326851
42	199999999999999999999989872438352903919476517967827644
43	1999999999999999999999968109588225979869129491956656
44	199999999999999999999999999999992027397056494967282372989148
45	199999999999999999999999999999998006849264123741820593247286
46	1999999999999999999999999999999501712316030935455148311821

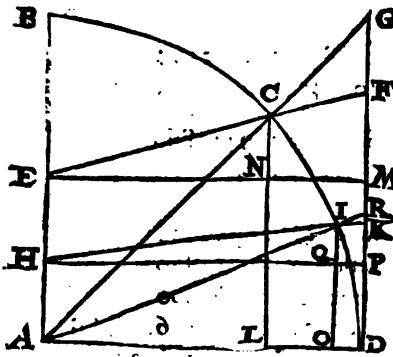
Hic est Subtentarum Canon; ad cuius usum declarandum, addo unum atque alterum exemplum. Esto radius circuli Quadrantis dati partic. 100000, 00000, 00000, 00000, 00000; ejusq; arcus ultimus ex secundo Canone ²⁶⁴¹⁵⁴⁵⁶ Quadrantis, respondens bisectioni radii & Quadrantis 28². & numero circulorum radii. Ex Canone subtentarum datur subtensa 28² partic. 199999, 99999, 99999, 96575, 79310, 93652, 548⁹, quarum diameter est 200000, 00000, 00000, 00000, 00000; 00000; 00000; Ejus semissis 99999, 99999, 99999, 98287 — est sinus complementi ultimi arcus. Subtentra vero immediatè præcedens est particul. 199999, 99999, 99999, 86303, 17243, 74610, 196²; Ejus & Diæmetri differentia est particul. 13696, 82756, 25589, 803¹⁰; hujusque latus quadratum particul. 1703, 34463, 41372, 77380, subtensa est arcus dupli, Ergo hujus semissis particul. 58516, 72317, 06863, 8690 est sinus rectus arcus ultimi.

Aliud exemplum; sit dati Quadrantis radius partic. 100000, 00000, 00000, 00000, 00000; 00000, 00000, 00000, 00000, ejusq; ultimus arcus ex secundo Canone ¹²³⁷²⁰⁸³³² Quadrantis, respondens bisectioni Quadrantis 45. & numero circulorum radii. Detur quoque 45 subtensa ex subtentarum Canone: particul. 199999, 99999, 99999, 99999, 99999, 98056, 41237, 41820, 59324, 7286; quarum Diameter est 200000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000; 00000; 00000; Hujus semissis particul. 99999, 99999, 99999, 99999, 99999, 99003 est — sinus complementi arcus ultimi. Subtentra vero proxime præcedens est particul. 199999, 99999, 99999, 99999, 99999, 99999, 99999, 924657, 39705, 64949, 67282, 37298, 9148; cuius & Diæmetri differentia est particul. 72716, 02943, 50593, 27176, 27010, 852; & differentia latus quadratum particul. 89189, 43334, 90206, 76626, 66856, 7119. est subtensa arcus dupli. Ergo semissis hujus 44644, 71677, 45104, 88313, 33428, 3564 est sinus rectus arcus ultimi.

Et hi quidem Canones sunt: quodruma admittculo, dicet Theorematis nostri veritatem experiri in Radiis majoribus. Nos autem contenti erimus duobus exemplis, nimirum radii particul. 100000, 00000, 00000; 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, quo nos primum utemur.

Terium exemplum radii particul.

100000, 00000, 00000, 00000, 00000, 000.



Repetatur præcedens diagramma, sitque A B
radius particul. 100000, 00000, 00000, 00000,
00000, 000, & A E ultima pars radii earundem
37252, 90298, 46191, 40625: Item D C arcus
ultimus Quadrantis $\frac{1}{6} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{6}$, & C L ipsius sinus
rectus in mensura Radii particul. 58516, 72317,
06863, 8690 — & E N complementi sinus
earundem 99999, 99999, 99999, 9828 —

Subducatur primum ex sinu

$$\begin{array}{l} CL \quad 5851672317068638960 \\ LN \quad 37252902984619140625 \end{array}$$

$$NC \text{ erit } 2126382018606724627$$

Quare per 4^m Sexti Euclidis

$$\begin{array}{ll} \text{Ut } EN & 99999999999999999828 \\ \text{ad } NC & 2126382018606724627 \\ \text{Ita } EM & 100000000000000000000 \\ \text{ad } MF & 2126382018606724664. \text{ Cui si} \\ \text{addas } DM & 37252902984619140625 \end{array}$$

$$Erit DF \quad 58516723170686387265$$

Biscentur porro A E & arcus D C, eritque A H 186264514923095703125, Item
D I $\frac{1}{3} \cdot \frac{1}{6} \cdot \frac{1}{9} \cdot \frac{1}{12}$ Quadrantis, ejusque rectus sinus I Q 2925836158534319360, & comple-
menti sinus H Q 999999999999999999957 —

$$\begin{array}{ll} \text{Auferatur verò & hic ex sinu } IO, & 2925836158534319360 \\ \text{A H} & 186264514923095703125 \end{array}$$

$$\text{reliqua erit } QI \quad 1063191009303362328$$

Quare per 4^m Sexti Euclidis

$$\begin{array}{ll} \text{Ut } HQ & 999999999999999999571 \\ \text{ad } QI & 1063191009303362328 \\ \text{Ita } HP & 100000000000000000000 \\ \text{ad } PK & 1063191009303364332. \text{ Cui si} \\ \text{addas } DP & 186264514923095703125 \end{array}$$

$$Erit DK \quad 292583615853431936325.$$

Itaque & hic recta HDK bifecat DF in K. Estenim

$$\text{Ut } DF \quad 58516723170686387265,$$

$$\text{ad } DK \quad 292583615853431936325.$$

Ita arcus D C ad arcum D I.

Quare per elementum præmissum DF est æqualis DC, & DK est æqualis DI.
Quod erat ostendendum.

Postremum exemplum radii particularum.

Retineatur & hic alitera præmissum, sitque A B radius particul. 100000, 100000,
00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, & A E pars radii ultima earundem

28421, 70943, 04040, 07434, 84497, 07031, 25; item D C ultimus arcus Quadrantis $31:8417:0883:2$, ejusque sinus rectus C L in mensura Radii partic. 44644, 71677, 45104, 88313, 33428, 3564; & complementi E N 99999, 99999, 99999, 99999, 99999, 9900.

Auferatur primum ex sinu

$$\begin{array}{r} CL \quad 44644716774510488313334283564 \\ LN \quad 28421709430404007434844970703125 \\ \hline \end{array}$$

$$\text{relinquitur NC} \quad 28223007344106480878489312860$$

Itaque per 4 Sexti Euclidis

$$\begin{array}{rl} Ut E N & 99999999999999999999999999999999900 \\ ad NC & 16223007344006480878489312860 \\ Ita EM & 1000000000000000000000000000000000000 \\ ad MF & 16223007344106480878489312876. Cui si \\ addas DM & 28421709430404007434844970703125 \\ \\ Erit DF & 44644716774510488313334283579125. \end{array}$$

Divide vero A E & arcum D C in partes duas aequales, erit A H particular. 14210, 85471, 52020, 03717, 42248, 53515, 625; item D I 7036874417766 , Quadrantis, ejusque sinus rectus I O 22322, 35838, 72552, 44156, 66714, 1787, & complementi sinus H Q 99999, 99999, 99999, 99999, 99999, 9975.

Subducatur vero & hic ex sinu

$$\begin{array}{r} IO \quad 22322358387255244156667141787 \\ AH \quad 142108547152020037174224853515625 \\ \hline \end{array}$$

$$\text{reliqua erit Q I} \quad 8111503672053240439244656436$$

Quamobrem per 4 Sexti Euclidis

$$\begin{array}{rl} Ut HQ & 999999999999999999999999999999975 \\ ad QJ & 8111503672053240439244656436 \\ Ita HP & 1000000000000000000000000000000000 \\ ad PK & 8111503672053240439244656438. Cui \\ si addas DP & 142108547152020037174224853515625 \\ \\ Erit DK & 223223583872552441566671417895625. \end{array}$$

Ergo & hic recta H IK bisecat D F in K. Est enim

$$\begin{array}{rl} Ut DF & 44644716774510488313334283579125 \\ ad DK & 223223583872552441566671417895625 \end{array}$$

Ita arcus D C ad arcum D I.

Quare per praecedens elementum D F est aequalis D C, & D K est aequalis D I. Quod erat demonstrandum.

Atque ita Theorematis nostri veritas luculenter demonstrata est. Sequuntur jam porismata, quae ex Theoremate haud aliter quam rivilii ex fonte suo derivantur: Ex quibus de praestantia atque utilitate ipsius Theorematis judicare promptum erit; Imprimis de multiplici ipsis Tangentis abscissis, quae vera est πr^2 , veterumq; πr^2 , totiusq; Cyclometriae fundamentum unicum. Enim vero haec ipsa linea cum peripheria sua affinitatem habet tantam, ut si ambitiosa & recta discrimen excipias, altera alterius naturam induisse videatur. Ut enim peripheria C D major est sinu suo C L, & minor tangente sua D G; ita etiam tangens abscissa D F, major est eodem sinu C L, & minor tangente D G: idque etiam ita est cum peripheria D C, & pars radii A E continuè bisecantur.

Sectando ut peripheria C D eodem modo se habet ad peripheriam dimidiam D I, quo pars radii A E ad partem dimidiem A H; ita etiam tangens abscissa D F se habet ad tangentem abscissam D K, ut pars radii A E, ad partem dimidiem A H; & sic quoque est in con-

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continuis peripheriaz DIC, & pars radii A E bisegmentis. Itaque non est dubium, quia altera alteri aequalis sit, sicutque in dato circulo nam si inaequales essent, nequam hæc fierent quæ diximus.

Erit vero etiam sinus & tangentes circa circuli contactum peripheriaz suis sunt aequales in dato circulo, quemadmodum 4^o Theoremate ostendimus, magnum tamen est inter hos, & tangentem abscissam discrimen. Nam tangentis abscissa peripheriaz naturam prorsus refert, ut modo probavimus: sinus autem & tangentes referre eam nunquam possunt, quia omnis sinus absolute perephera sua semper est minor, & omnis tangentis major.

Secundò quoniam Sinus & Tangentes ad circuli contactum peripheriaz suis primum aequales evadunt in dato circulo, usum quidem habent in circuli dimensione quæ fit per numeros, non autem in illa quæ absolvitur per lineas: ratio est, quod ejusmodi sinum aut tangentem peripheriaz suæ adscribere non licet. Contra quia tangentis abscissa, peripheriaz naturam refert, etiam cum quadrantis dimidii intervallo à punto contactus distat, non modò utrius dimensioni apta est, sed multò ante diametri & peripheriaz rationem in numeris exhibit, quam sinus aut tangentis.

Verum quia hæc aliaque quæ huc faciunt, ex Theorematibus nostri porismatis maximè erunt perspicua, subiicio porisma ipsa.

P O R I S M A . I.

Hinc licet primò cujuscunque circuli propositi peripherie aequalem rectam describere. Quarta enim proportionalis radii parti ultima, tangentique abscissa & radio, est aequalis circuli propositi quadranti, & ipsius quadrupla toti circulo.

Hic primus est usus tangentis abscissæ, viz. quod ipsius beneficio, cuius circulo proposito aequalis recta describatur. Cujus Problematis à veteribus diu mutuorumque est quæ sita, nuncquam inventa. Dinostratus enim hinc fini exegita verat regula, ut Archimedes ordinatam in utramque tangentem lineam inutilem, quod ex ipsorum principiis describi non posset. De Dinostrata linea res nota est ex Sporo, Pappo, atque aliis, & à nobis infra, volente Deo, demonstrabitur.

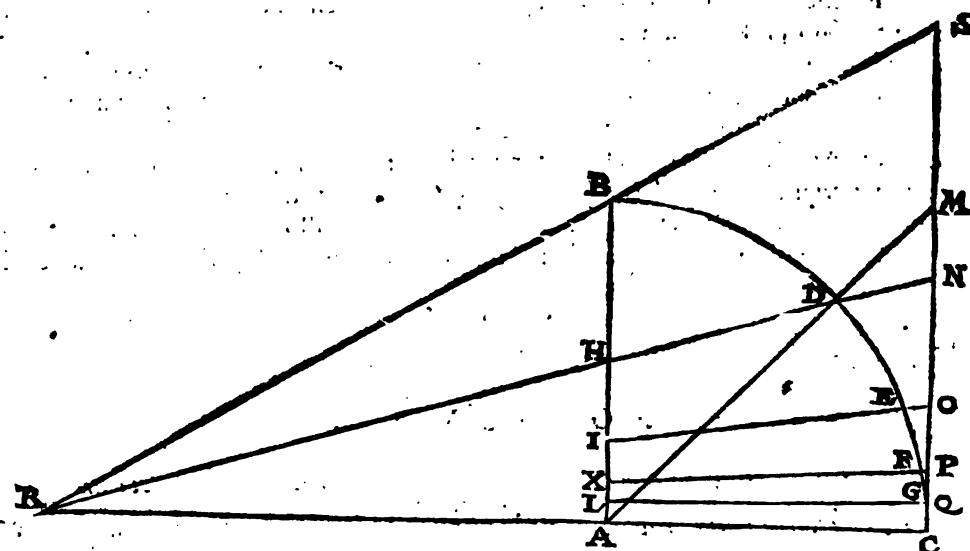
Ex Archimedea constabit, si duximus instituatur per numeros. Nam si radius circuli, in quo prima helicis conuersio absolvitur statuerit particul. 100000, recta eidem peripheriez aequalis, erit particul. 628318, ac suo loco ostendetur. Cum vero per Archimedem recta terminum voluta contingens, abscindat ab infinita, quæ ex circuli centro per prium quadrantis terminum ducitur, rectam eidem circulo aequali; necesse est eandem lineam abscissam esse earundem particul. 628318, angulumque quem linea abscissa subtendit, ex Canone Tangentium grad. 80 57 25. Jam si ex Archimedais principiis recta sit describenda circuli propositi peripheriez aequalis, oportet contingenciam ita ducere, ut abscissa hunc ipsum angulum exactè subtendat. Nam si angulum subtendat uno tantum primo scrupulo minorem, abscissa linea est particul. 636654 multo minus justa: si angulum subtendat uno scrupulo majorem, eadem particul. erit 639006, justa multo maior. Atqui cum ex Archimedais principio, contingens sic duci nequeat ut dictum angulum exactè subtendat, unius recta per eam obtineri potest, proposita peripheriez aequalis.

Nos itaque prissi aperiimus prius cuicunque circulo proposito, aequalem rectam describendi. Verus enim illa, quæ utitur ratione diametri & peripheriez tripla & sequente, nec veritatis sue monumentum habet à se, nec omni circulo proposito congruit, sed tantum omnium qui dari possunt minimis. Nostra econtra & robur, veritatis à se accipit, & cuiusvis circulo proposito dimetienda apta est; Itaque ea ipsa est quæ tot seculis, totque à Geometris summo studio, ac labore quæ sita fuit, & suæ primum, summum Dei beneficio est, inter yeara, & præmisso porismate expressa. Est autem ipsius arithmetica hæc.

Este circuli propositi quadrans A B C, cuius peripheria B C bisectetur in D, radiusque A B in H: & à punto bisectionis radii H, per bisectionis punctum Quadrantis D,ducatur recta H Q N in tangentem Quadrantis dimidii D M, quæ ex tangentem suæ M abscindat tan-

Cyclometriae Liber I.

tangentem CN. Dico quartam proportionalem ultima pars radii AH, tangentis abscissae CN, & radio AB, aequalem esse quadranti ABC, si radius ponatur particul. duxat 10, vel 100, utpote circuli omnium minimi. Nam per demonstrata Pappi est, Ut AH ultima pars radii ad DC ultimum arcum Quadrantis, ita AB radius ad BC Quadrantem.



Quoniam autem per Theorema præmissum tangens abscissa CN est aequalis ultimo arcui DC; Per 7 Quinti Euclidis est, Ut AH ultima pars radii ad CN tangentem abscissam ultimo arcum Quadrantis aequalem, ita AB radius, ad quartam proportionalem, Quadranti BC aequalem. Inventâ igitur quarta proportionali ultima pars radii AH, tangenti abscissa CN & radio AB, inventa quoque est recta circuli propositi Quadranti BC aequalis, quæ postulatur.

Invenitur autem ea promptè, si recta HDN, & radius AC continentur, dum se se intersectent in R; & ex R per B terminibus radii AB recta RBS ducatur in tangentem CS. Tunc enim per 10^m Sexti secatur tangens CS eodem modo in N, ut radius AB in H, adeoque AH AB sunt proportionales CN CS & per mutando, AH CN, proportionales sunt AB CS. Itaque Tangens CS est quarta proportionalis ultima pars radii AH, tangenti abscissa CN, & radio AB; eademque est aequalis Quadranti BC, & ipsius quadruplica toti circulo. Descripta igitur est recta aequalis circulo proposito. Quod erat faciendum.

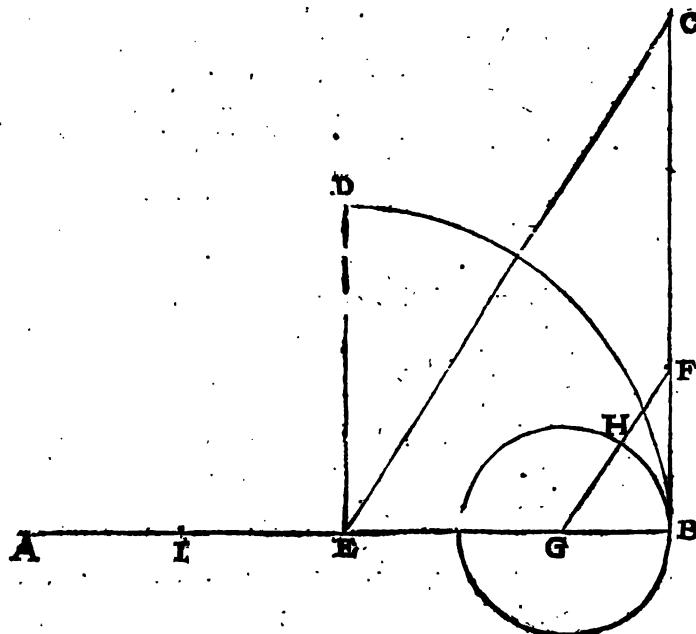
Quod si vero proponatur circulus major, continuare oportet peripheria Quadrantis & radii bisectionem, pro circuli dati magnitudine, atque ultimo arcui aequali tangentem abscindere, & in ceteris procedere ut supra; ita enim licet cuivis circulo proposito aequali rectam describere, In exemplo, cum circulus proponitur minimus, quadrans ipsius BC, & radius AB, in duas quatuorve partes aequales secantur, ultimoque arcui CD vel CE aequalis abscinditur tangens CN, vel CO; atque ita datur CS, aequalis quadranti BC, cum ratione diametri & peripherie Archimedea, tripla & sesquiseptima, quæ minimo circulo mensurando sufficit, non autem majoribus. Vetus si circulus proponatur minimo paulò major, oportet ipsius quadrantem & radium dividere in partes aequales octo, ultimoque arcui CF abscindere aequali tangentem CP; sic enim datur CS aequalis Quadranti BC, cuim ratione diametri & peripherie Ptolemaica, quæ media est inter triplam sesquiseptimam, & triplam super partientem decem septuagesimas primas. Quod si vero & hoc circulo proponatur paulò major, ipsius quadrans BC & radius AB dividendi sunt in partes aequales sexdecim, ultimoque arcui CG, aequalis tangens abscindenda CQ, hinc enim datur CS aequalis Quadranti BC, cum ratione diametri & peripherie ut 10000 ad 31416; qua Viri magni Georgius Purbachius, & Franciscus Vieta sunt: m. Licet autem bisectionem peripherie Quadrantis & radii hoc modo continuare quotieslibet, adeoque cui-

cuius circulo proposito æqualem rectam ducere. Cujus problematis constructio, jam totos bis mille & sexingentos annos à Magnis Viris quæsita, à nobis primùm, Dei Opt. Max. beneficio, inventa, jam in omnium conspectum sistitur.

P O R I S M A I I.

Secundò cuiusvis recte data describi potest æqualis circuli peripheria, si prius circuli cujusvis quadranti æqualis recta descripta fuerit. Quarta enim proportionalis hinc recte, ratioque circuli, ex recte data quadranti, est radius circuli postulati.

Hoc porisma est superioris conversum, sibiisque etiam suum à superiore accipit, ut sequens demonstratio docet. Sit enim recta A B data, cui æqualem circuli peripheriam describere oporteat; sitque prius cujusvis circuli Quadranti descripta æqualis recta, per porisma præcedens; exempli gratia in nostro Diagrammate, recta B C æqualis Quadranti D B. Dico quartam proportionalem rectæ B C, radio E B, & A I (quæ est quarta pars datæ A B) esse radium circuli postulati. Nam per demonstrata Pappi est, ut B C recta



Quadranti D B æqualis, ad E B ipsius radius; ita A I (quarta pars A B datæ) æqualis circuli postulati quadranti, ad ipsius radius. Inventa igitur quarta proportionali rectæ B C, radio A B, & A I quartæ parti ipsius A B datæ, obtinetur radius circuli ipsi A B datæ æqualis.

Quarta autem proportionalis dicta promptè invenitur, si ex B C abscindatur B F, æqualis A I, rectæque E C parallela ducatur G F. Quia enim triangula E B C & G B F ex fabrica sunt similia, per 4^m sexti est,

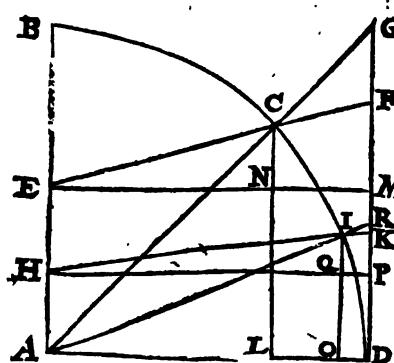
Ut B C ad E B, ita B F ad G B.

Itaque G B est quarta proportionalis B C, E B, & B F; eademque est radius circuli H B A rectæ A B datæ æqualis. Quamobrem rectæ A B datæ descriptus est circulus H B H æqualis. Quod erat faciendum.

P O R I S M A I I I .

Tertiò cujuslibet circuli perimeter dari potest in data mensura diametri. Nam ut sinus rectus complementi ultimi arcus est ad differentiam sinus recti ejusdem arcus & ultime partis radii; ita radius ad differentiam perimetri arcus ultimi; & ultime partis radii, que enim ultima pars radii, componit perimetrum arcus ultimi. Sed ut ultima pars radii est ad perimetrum arcus ultimi, ita radius ad Quadrantis circuli dati perimetrum. Ergo hujus quadruplus est dati circuli perimeter.

Exposita Dimensione circuli Geometrica qua sit per lineas, sequitur Arithmetica quæ absolvitur calculo. Hujus suminam complexi sumus præcedente porismate, cujus partes sunt duæ. Prima calculum proponit perimetri arcus ultimi; Altera Quadrantis perimetri.



Utriusque fundamentum ex sequenti demonstratione manifestum est. Repetatur diagramma, quo supra in demonstratione Theorematis usi sumus; sitque B C D quadrans circuli, cujus radius sit datum particul. 100. Quæritur ejusdem perimetrum in data mensura radii. Quia radius A B est particul. 100, A H ipsius Quadrans ex fabrica est particul. 25: item arcus D I, quarta pars Quadrantis B C D partic. 90, est earundem 22 $\frac{1}{2}$; ejusque sinus rectus I O particul. 38 $\frac{1}{2}$ qualium A B est 100; & complementi H Q earundem 92 $\frac{1}{2}$. Ergo I Q differentia sinus I O, 38 $\frac{1}{2}$, & Q O ultimæ partis radii 25 est 13 $\frac{1}{2}$. Hinc datur ex præmisso Theoremate D K, perimetrum arcus ultimi D I, particul. 39 $\frac{1}{2}$. Nam ut H Q sinus complementi arcus ultimi D I partic. 92 $\frac{1}{2}$ ad I Q 13 $\frac{1}{2}$ differentiam sinus I O & ultimæ partis radii O Q; ita H P 100 ad D K 14 $\frac{1}{2}$ differentiam perimetri ultimi arcus D K, & ultimæ partis radii D P; quæ cum ultima parte radii D P 25 componit D K perimetrum arcus ultimi 39 $\frac{1}{2}$.

Atqui per 15 Quinti Euclidis,

Ut A H ultima pars radii 25 se habet ad D K perimetrum arcus ultimi 39 $\frac{1}{2}$, ita A B radius 100 ad perimetrum Quadrantis B C D 157 $\frac{1}{2}$, cuius duplus 314 $\frac{1}{2}$ est perimetrum semi-circuli, puta si radius sit particul. 100; vel circuli perimetrum, si radius sit particul. 50.

Atque hæc est illa diametri & peripheriæ ratio tripla & sesquiseptima, qua Euclides septuaginta quinque annis ante Archimedem usus est; quamque Archimedes duplice, eaque operosa demonstratione comprobavit. Nam per regulam auream est

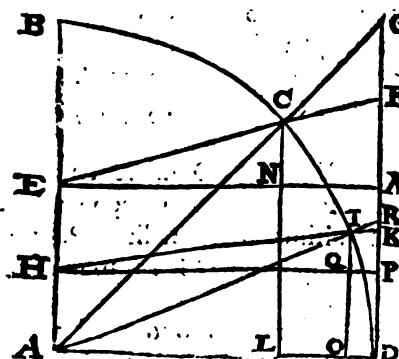
Ut 100 ad 314 $\frac{1}{2}$, ita 7 ad 22.

Quæ ipsa quoque invenitur ex prima bisectione Quadrantis B C D in C, & radii A B in E. Tunc enim D F est perimetrum ultimi arcus D C. Atqui jam ante in primo Theoremati nostri exemplo ostendimus perimetrum D F esse particul. 78 $\frac{1}{2}$ fere, quarum A B radius est 100; itaque perimetrum dupli arcus, id est quadrantis B C D est earundem 157 $\frac{1}{2}$, omnibus modis ut supra.

Cæterum quia ista diametri & peripheriæ ratio, non convenit omnibus circulis, sed tantum minoribus, subjicio exempla circulorum majorum; ex quibus apparebit, quod de minoribus jam demonstratum est, verum esse in majoribus omnibus, etiam in vastissimis.

Secun-

Secundum exemplum radii particul. 1000.



Repetatur præcedens diagramma, in quo radius AB sit particul. 1000: & AH ultima pars radii earundem 125, nimur pars radii octava, & DI ultimus arcus grad. 11 15; ejusque sinus rectus IO ex Canone Sinuum partic. 1950, & complementi HQ 9807, qualium AB radius est 10900 (augmentus enim radii datum uno circulo, ut sinus dicti accurate habeantur quod etiam in præmissis exemplis fecimus.) Itaque IQ differentia sinus IO, & ultimæ partis radii QO est 700. Hinc jam est ex præcedente Theoremate,

Ut HQ sinus rectus complementi ultimi arcus 9807, ad differentiam IQ 700, ita HP 10000 ad

KP 713, differentiam perimetri ultimi arcus DK, & ultimæ partis radii DP; Quæ cum ultima parte radii DP 125, componit perimetrum arcus ultimi DK 1963.

At per 15^m Quinti Euclidis,

Ut AH ultima pars radii 125, est ad DK perimetrum arcus ultimi 1963, ita AB-radius 1000, ad perimetrum Quadrantis BCD 1570: Cujus duplus est perimeter semicirculi, & quadruplus circuli perimeter. Itaque semicirculi perimeter est partic. 3141⁸ quarum radius ponitur 1000. Vel si diameter ponatur particul. 1000, circuli perimeter est earundem 3141⁸. Ratio autem diametri 1000, ad perimetrum 3141⁸ est media inter triplam sesquiseptimam, & triplam superpartientem decem septuagesimas primas. Quam Ptolemæus accuratiorem esse testatur, ratione tripla sesquiseptimâ, qua Euclides & Archimedes usi sunt. Vide *myda. m. 76g.* librum V I. cap. V I I.

Verum quia & hæc ratio tantum deservit circulis, quorum diametri in particulas 1000 commode dividuntur, non autem majoribus, subjicio tertium exemplum radii particular. 10000.

Tertium exemplum radii particul. 10000.

Manente superiore diaphra; sit AH ultima pars radii $\frac{1}{2}$, adeoque particul. 625, quarum AB radius est 10000: item DI ultimus arcus quadrantis, sit $\frac{1}{16}$; ejusque sinus rectus IO ex Canone Sinuum particul. 9801, & complementi HQ 99518: qualium AB ponitur 100000: Denique IQ differentia sinus IO & ultimæ partis radii QO particularum 3551. Hinc datur DK perimeter ultimi arcus DI 9818. Nam per præmissum Theorema

Ut se habet HQ sinus complementi arcus ultimi 99518, ad differentiam IQ 3551, ita HP 100000 ad KP 3568, differentiam perimetri ultimi arcus DK, & ultimæ partis radii DP, Quæ cum ultima parte radii DP conficit ultimi arcus perimetrum DK 9818.

At per 15 Quinti Euclidis,

Ut AH ultima pars radii 625, ad DK perimetrum arcus ultimi 9818 ita AB radius 1000 ad perimetrum Quadrantis BCD 1570⁸. Cujus duplus 31416 proxime, est perimeter semicirculi, & quadruplus perimeter circuli.

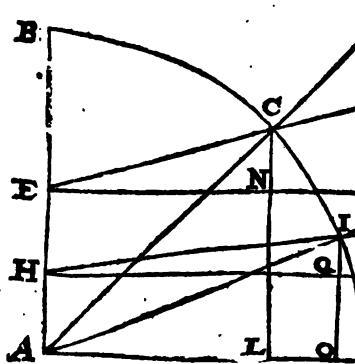
At si diameter propositi circuli statuatur particul. 10000, perimeter ipsius est 31416. Quæ ratio accuratior est illa quam Ptolemæus prodidit, Ut 1 ad 3. 8. 30, hoc est ut 1000 ad 31418. Ut non sine causa eandem proposuerit Magnus Vir Georgius Purbachius in tractatu suo de Sinibus; & Illustris Vir Franciscus Vista eadem usus sit in describenda recta circuli dati peripherie æquali.

At quoniam nec hæc mensurandis circulis majoribus sufficit, addimus quartum exemplum ex magno Canone Rhetici, Radii scz. particul. 10000000000.

Quartum exemplum radii particul. 10000000000.

Sic itaque in eodem schemate radius A B particul. 10000000000, & A H pars radii earundem 9765625; item I D ultimus arcus Quadrantis 15339801; ejusque sinus rectus I O 15339801 in mensura radii particul. 10000000000; & complementi 999998234. Denique sinus recti I O, & ultimae partis radii Q O differentia I Q, particul. 5574176. His datis, per præmissum Theorema est.

Ut H Q sinus rectus complementi ultimi arcus 999998234 ad I Q differentiam sinus recti ultimi arcus, & ultimæ partis radii 5574176, ita H P radius 10000000000 ad K P 5574183, differentiam perimetri arcus ultimi D K, & ultimæ partis radii D P. Quæ differentia si ad ultimam partem radii D P adjiciatur, componitur D K 15339808 perimeter ultimi arcus D I. At per 15^m Quinti Euclidis.



Ut A H ultima pars radii 9765625 est ad D K ultimi arcus perimetrum 15339808; ita A B radius 10000000 ad perimetrum Quadrantis B C D 1570 7963. Cujus duplus 31415926 est perimeter semicirculi.

Verum si diameter ponatur particul. 10000000, M circuli perimeter est 31415926. Quæ diametri & perimetri ratio, accuratissima est omnium quæ ex magno Canone Triangulorum deducuntur. Ex hypothesi enim radii particul. 10000000000, (quem idem Canon supponit) perimetrum dare licet qui respondeat radio particul. 10000000, non autem radio partic. 10000000000. Nam cùm ratio radii ad perimetrum quadrantis colligatur ex ratione partis

radii ultimæ ad perimetrum arcus ultimi; perimeter autem arcus ultimi tribus saltem notis, à radii dati notis deficiat; oportet etiam quadrantis perimetrum totidem notis à radii notis deficere, ut consequentes magnitudines, antecedentibus (& quidem $\alpha\beta\gamma\mu$) respondeant. Quamobrem ut hic, ita etiam in reliquis exemplis omnibus, numero notarum perimetri arcus ultimi, semper quadrantis perimetri notarum numerus æqualis est pendens; ita enim ratio radii ad perimetrum quadrantis, respondebit exacte rationi ultimæ partis radii ad ultimi arcus perimetrum.

At quia nec hæc diametri & peripheriarum ratio, majoribus circulis convenit, adjicio quintum exemplum radii particul. 100000, 00000, 00000, 00000, 00000, 000.

Quintum exemplum radii partic. 100000, 00000, 00000, 00000, 00000, 000.

Repetatur præcedens figura, ponaturq; A B radius particul. 100000, 00000, 00000, 00000, 000; et A H ultima pars radii earundem 37252, 90298, 46191, 4062; item D I ultimus Quadrantis arcus 26435416; & I O ejusdem sinus rectus in mensura radii particul. 58516, 72317, 06863869 --- & H Q complementi sinus earundem 99999, 99999, 99999, 9828 --- tandemq; sinus recti I O & ultimæ partis radii Q O differentia I Q, particul. 21263, 82018, 60672, 4627. Hinc per præmissum Theorema est.

Ut H Q sinus rectus complementi ultimi arcus 99999, 99999, 99999, 9828 --- ad I Q differentiam 21263, 82018, 60672, 4627, ita H P radius 100000, 00000, 00000, 00000, ad K P 21263, 82018, 60672, 4664 differentiam perimetri arcus ultimi D I, & ultimæ partis radii D P. Quæ differentia cum ultima parte radii, componit D K 58516, 72317, 06863, 8726 perimetruin arcus ultimi D I, neglecta fracione 1, quia sinus ultimi arcus I O, tantum deficit ab ultima parte radii A H.

Ergo per 15^m Euclidis

At A H ultima pars radii 37252, 90298, 46191, 4062 ad D K 58516, 72317, 06863, 8726

8726 perimetrum arcus ultimi; ita A B radius 100000, 00000, 00000, 000, ad 15707, 96326, 79489, 6619, perimetrum Quadrantis. Cujus duplus 31415, 92653, 58978, 3238 --- est perimeter semicirculi.

At si diameter statuatur particul. 100000, 00000, 00000, 000, circuli perimetru est earundem 31415, 92653, 58978, 3238 ---

Quæ ratio perimetri ad diametrum multò accuratiæ est illâ quam Clarissimus Ludolphus à Colle, in opere suo Cyclometrico, ex ejusdem arcus inscripta & circumscripta demonstravit, nimirum ut 100000, 00000, 00000, 0 ad 31415, 92653, 58978, 32 minorem justâ; & 31415, 92653, 58978, 33 justâ majorem. Ultraque enim duabus ultimis notis deficit à nostra. Apparet itaque verum esse quod supra diximus, rationem diametri & peripherie citius, accuratiusque obtineri per ultimi arcus tangentem abscissam, hoc est per nostram Cyclometriam, quâm per ejusdem arcus inscriptam & circumscriptam: ideoque nostram Cyclometriam, Archimedæa (qua Ludolphus usus est) negligamus, magisque compendiofam esse. Nam quod Archimedæam nonnulli nostræ præferendam esse existimant, quod ea ultimam perimetri notam perpetuò concludat intra duos terminos majorem & minorem, nostra verò hoc faciat nunquam: error est, quia & nostra hoc ipsum cum Archimedæa perpetuò facit. Enimvero Cyclometria nostra ultimam notam semper dat exactè, quemadmodum in præcedente exemplo demonstravimus: est itaque sine fractione, justâ semper minor; & cum fractione, vel cum unitate, perpetuò justâ major. In exemplo, si detur Diameter partic. 1000000, perimetru major est quâm 31415926, & minor quâm 31415927, vel etiam quâm 31415926.⁵. Item si diameter detur partic. 100000, 00000, 00000, 000, perimetru major est quâm 31415, 92653, 58978, 3238, & minor quâm 31415, 92653, 58978, 3239. Atque ita in cæteris; si modò numerus notarum perimetri Quadrantis, æqualis sit numero notarum perimetri arcus ultimi.

Quoniam vero neque ista Diametri & Perimetri ratio, locum habet in circulis majoribus, addo sextum exemplum radii Vastissimi, particul. 100000, 00000, 09000, 00000, 00000, 00000, 00000, 00000.

Postremum exemplum radii particular.

100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000.

Esto A B radius in adjuncto Diagrammate partie. 100000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000; erit A H ultima pars radii earundem 28421, 70943, 04040, 07434, 84497, 07031, & D I ultimus arcus Quadrantis _{3, 18412, 08881, 1}, ejusque sinus rectus I O in mensura radii particul. 44644, 71677, 45104, 88313, 33428, 3564; & complementi H Q sinus 99999, 99999, 99999, 99999, 99999, 99999, 9900; item recti sinus I O, & ultimæ partis radii Q O differentia I Q particul. 16223, 00734, 40064, 80878, 48931, 2860. Ergo per Theorema præmissum est.

Ut H Q sinus rectus complementi ultimi arcus 99999, 99999, 99999, 99999, 99999, 99999, 9900, ad differentiam I Q 16223, 00734, 40064, 80878, 48931, 2860, ita H P radius 100000, 00000, 00000, 00000, 00000, 000, ad K P 16223, 00734, 40064, 80878, 48931, 2876, differentiam perimetri arcus ultimi D K, & ultimæ partis radii D P. Quæ differentia cum ultima parte radii componit perimetrum arcus ultimi D K particul. 44644, 71677, 44104, 88313, 33428, 3579, omissa fractione ultimæ partis radii $\frac{1}{2}$, quia sinus ultimi arcus F O, tantundem deficit ab ultima parte radii.

Itaque per 15 Quinti, ut A H ultima pars radii 28421, 70943, 04040, 07434, 84497, 07031 ad D K 44644, 71677, 44104, 88313, 33428, 3579; ita A B 100000, 00000, 00000, 00000, 00000, 000, ad 15707, 96326, 79849, 66192, 31321, 6916.⁵ Cujus duplus 31415, 92653, 58979, 32384, 62643, 3832⁵ est semicirculi perimeter. Itaque ~~est~~ ^{est} 31415926535897932384626433832⁵ ⁵ ~~est~~ ^{est}.

Et hæc quidem exempla sufficiunt illuftrando Thœorematis nostri porismati tertio; eademque perspicuè docent quomodo in terminis multo majoribus, ratio Diametri ad peri-

perimetrum definiri possit, si modò Canon Subtenarum ad plures particulas sit subduc-tus. Cujusmodi est quem magnus Logista Ludolphus à Collen suppeditavit ad Diametri cir-culos 75. Verùm quia tam infiniti numerorum anfractus, nec usum habent ullum, nec ad Cyclometriæ perfectionem illo modo faciunt, non libet nobis ultra λατενολογιαν. Omnino enim nos cum Medicorum principe statuimus, πάντας τὰς ἀποδιημέτρους τὰς εἰς τὴν βιωφολίαν, τὰς δὲ διανομέτρους. Ideoqué numeris quos supra exposuimus, contenti sumus.

Porrò et si ex iis quæ hucusque demonstrata sunt, cuivis judicare promptum fit, quan-tum Cyclometria nostra super Archimedæam caput offerat, ut tamen ipsa rei veritas sit magis conspicua, exponam paucis, quid inter nostram, & Archimedæam interficit. Archi-medes tertia propositione μηδέποτε κύκλον demonstrat cujusvis circuli peripheriam rationem habere ad Diametrum minorem tripla sesquiseptima, & majorem tripla superdecupar-tiente septuagesimas primas. Unde infert justam peripheriæ & Diametri rationem intra terminos illos conclusam esse. Hoc quamvis adeo sit verum, ut qui negare audeat, ex Mathematicis Scholis tanquam άπορίαν eliminari mereatur: quia tamen Cyclometria ex eo ratiocinio deducta, crassior est quam Geometriæ subtilitas fert, non videtur abso-lutè Geometrica esse.

Manifestum enim est ex iis quæ supra demonstrata sunt perimetrum arcus ultimi esse ad ejusdem arcus sinum, ut idem arcus ad eundem sinum. Quomodo igitur ultimi arcus sinus Geometricè datur, ita quoque date oportet, arcus ultimi (adeoque & circuli ipsius) peri-phenetrum. Atqui Ptolemaeus libro magni operis I. cap. I X. ubi ex Hipparchi & Menelai sententia quantitates subtenarum Geometricè demonstrat, non cogit eas intra duos limi-tates majorem & minorem, sed determinat singulas in assumpta mensura diametri, exactè si rationales sint, vel ῥησίς si irrationales. Eadem itaque ratione peripheriæ Quantitas in assumpta mensura diametri danda est, non autem intra duos terminos concludenda.

Nam ut exemplo rem declarem, Si quis sinum semiquadrantis pronunciet majorem esse quam $\frac{7}{10}$, & minorem quam $\frac{3}{10}$, verum quidem dicet, sed ex arte sinum semiquadrantis non dabit; cum potius ex Ptolemaei doctrina pronunciare debeat, sinum semiquadrantis esse particul. 7071068 fere, qualium radius est 1000000. Atque ita etiam in dimensione cir-culi est procedendum. Nam si dati circuli peripheriam ex arte metiri libeat, non oportet cum Archimede pronunciare rationem peripheriæ ad diametrum inter $\frac{3}{7}\frac{1}{7}$ & $\frac{3}{7}\frac{1}{1}$ comprehensam esse, sed potius affirmare cum Ptolemaeo, circuli peripheriam esse partic. 3. 8. 30, qualium diameter est 1. lib. μηδέποτε. V I. cap. V I I. vel ex nostra doctrina accuratius, peripheriam circuli esse part. 31416, proxime, qualium Diameter est 1000.

Sed & alterum in Archimedæo ratiocinio animadvertendum est, viz. quod limites $\frac{3}{7}\frac{1}{7}$ & $\frac{3}{7}\frac{1}{1}$ latè nimis dissideant. Ex priore enim limite colligitur ratio Diametri ad peripheriam ut 10000 ad 31428 ---- ex altero ut 10000 ad 31408 ---- at quæ inter has est media scz. ut 10000 ad 31418 ----: haud satis est accurata. Supra enim in tertio nostro exemplo ostendum est diametro particul. 10000 deberi perimetrum particul. 31416 proximè; itaque peri-meter particul. 31418, non est justus. Atque hoc est quod observavit ante nos, Apol-lonius Pergæus magnus Geometra, qui non modò postulavit diametri & peripheriæ ratio-nem Archimedæa accuratiorem, sed ut Eutocius Ascalonita testatur, ἀνίδιξεν ποτε φερόμενον ισόγονον τὸ σύνορον μάκρον ἄγαν. Ideem fecit Philo Gadarenus, quem idem Euto-cius affirmit εἰς ἀνεργίας ἀριθμὸν ἄγαν τὸν ίππον Α'εγχυνόντας τοῦτο ζεψει τῷ τοῦ περι-

Sequitur vero tertium in Archimedæo ratiocinio notandum, nimirum quod ratio dia-metri & peripheriæ tripla & sesquiseptima, hoc est ut 7 ad 22, quæ tantum servit di-men-sioni circuli minoris, puta cujus diameter ponitur particul. 100, perperam propositione μηδέποτε κύκλον secunda majorum circulorum dimensioni adhibeatur. Licet enim ex ratione peri-

peripheria & diametri majoris, minoris quantitatem colligere, sed non contra ex ratione minoris, quantitatem majoris. In exemplo, ex ratione diametri & peripherie ut 10000, ad 31416 proxime, recte infertur ratio diametri & peripherie ut 100 ad 314. Est enim per regulam auream, ut 10000 ad 31416, ita 100 ad 314. Ex hac vero non sequitur illa, quia per eandem regulam est, ut 100 ad 314, ita 10000, ad 31400, quae minor est justa. Eodein modo ex ratione diametri & peripherie, ut 10000 ad 31416, sequitur ratio tripla sesquiseptima proxime; nam ut 10000 ad 31416, ita 7 ad 22 feret. At non ex ratione tripla & sesquiseptima sequitur ratio Diametri 10000 ad Perimetrum 31416; est enim ut 7 ad 22, ita 10000 ad 31428, quae particulis 12 illa est major. Itaque ne Cyclometria sit mendax, oportet vel ex ratione diametri & peripherie majoris data, inferre quantitatem minoris; vel circuli dati perimetrum ex praesenti porisimata determinare in data mensura diametri; utrumvis enim fiat, Cyclometria erit vera.

Sed haec quidem praecipua sunt quae in Archimedæo ratiocinio animadvertenda esse existimamus; ex quibus judicare licet de Cyclometria nostræ præstantia. Quæ enim in Archimedæo desiderantur, reperiuntur in nostra: & quæ demonstratione operosa ab Archimedæo adstruuntur, facili & perspicua à nobis expediuntur. Reliquum est ut in Cyclometria Dinostrati deinceps tentemus, quod in Archimedæo, Deo juvante, fecimus & perfecimus.

6. Si in dati circuli quadrante ab ultimo sectionis radii erecti puncto, recta ducatur in tangentem ultimo arcui aqualem, & ex centro quadrantis in dictæ tangentis terminum alia recta agatur priorem secans; perpendicularis à puncto sectionis in radium absindet basin. *περιγραφής* Dinostrati.

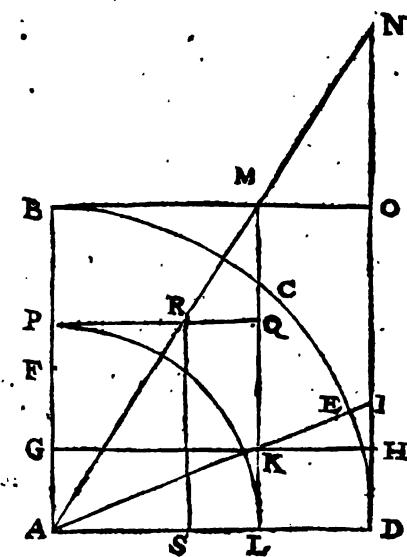
Inter lineas quæ Geometrarum scriptis celebrantur, duæ primum locum obtinent, Admirabilis & *περιγραφής*. Pappus admirabilem tribuit Menelao: *περιγραφής* vero idem Pappus cum Proclo attribuunt Dinostrato, Nicomedi, Hippiz. Conati autem sunt magni illi Viri *περιγραφής* describere per duos motus imaginarios, radii scz. & lineæ contra basin Quadrantis parallelæ; quæ dum motu *μελός* & *ινχεσίος* procedunt, radius quidem Quadrantis & parallela radium erectum percurrendo, quacunque earum communis sectio procedit, linea dicitur, quæ ab officio *περιγραφής* appellatur, quia scz. excogita fuit ad circulum quadrandum. Id vero inventum reprehendit Pappus quia principium petit. Cum enim potissimum ei fini comparatum sit ut punctum *περιγραφής* definit, idque prius evanescat quam inventum sit, neque ulla ratione ex Dinostrati principiis obtineatur, recte eam rejicit Pappus, ut inutilem, & quæ describi non possit.

Tentavit superioribus annis Doctissimus Clavius eandem describere per puncta radii: & paralleles sece intersecantium (quod tamen artificiū magnos illos Viros non latuit) sed conatu irrito: quia ut Sporus Nicenus animadvertisit, & Clavius ipse fateri cogit, ipsius *περιγραφής* finis eo modo nunquam deprehenditur.

Nos itaque primi aperimus viam terminum lineæ *περιγραφής* deprehendendi; eamique munimus demonstratione sequenti.

In adjuncta figura, esto circuli dati quadrans A B C D inscriptus quadrato A B O D, cuius peripheria B C D sit continuè bisecta, primum in C, secundò in E, bisectus quoque sit eodem modo radius A B, primum in F, secundò in G; Deinde per præmissum Theorema describatur recta D I, æqualis arcui ultimo E D. Tandem ex G ultimo bisectionis radii puncto agatur normalis GH in tangentem DI, & ex A centro quadrantis, mittatur alia recta AI in terminum tangentis D I, secans priorem GH in punto K. Dico A L partem radii AD quam absindet perpendicularis KL à puncto sectionis K in radium AD, esse basin *περιγραφής* Dinostrati.

Con-



Continuetur enim K L in M, & ex A centro ducatur recta A N per punctum M in tangentem D N, eruntque triangula A L M & A D N similia, propter rectos angulos ad L & D, communem ad A: ideoque per 4^m Sexti Euclidis.

Ut A L ad L M, ita A D ad D N.

Est autem D N æqualis quadranti B C D. Nam per 15 Quinti Euclidis.

Ut K L quarta pars radii, ad I D rectam æqualem quartæ parti Quadrantis B C D, ita L M id est A B radius, ad D N rectam æqualem Quadranti B C D. Ergo per 7 Quinti

Ut A L ad L M radium, ita A D radius ad quadrantem B C D: adeoque recta A L, radius L M, & quadrans B C D sunt continuæ proportionales.

Demonstravit verò Dinostratus basin *τετραγωνιζόμενος*, radius, & peripheriam Quadrantis continuæ proportionales esse. Quamobrem cum pars radii abscissa A L, radius A D, & peripheria B C D continue proportionales sint; sequitur partem radii abscissam A L esse basin *τετραγωνιζόμενος* dinostrati. Quod erat demonstrandum.

P O R I S M A.

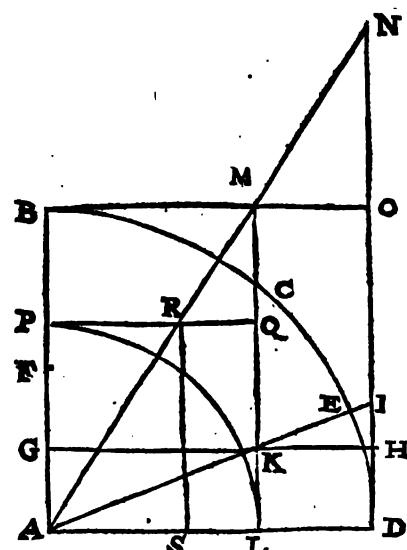
Itaque tertia proportionalis basi τετραγωνιζόμενος, & dati quadrantis radio, peripherie dicti quadrantis æqualis est.

Quia enim ex dinostrati demonstratis, basis *τετραγωνιζόμενος*, est ad radium, ut radius ad peripheriam Quadrantis; & ex nostri Theoremati *Διδάξει*, basis *τετραγωνιζόμενος* est ad radium, ut radius ad tertiam proportionalem; manifestum est peripheriam Quadrantis & tertiam proportionalem habere eandem rationem ad radium; atque adeò per 9. Quinti Euclidis peripheriam Quadrantis & tertiam proportionalem inter se æquales esse.

Apparet autem ex præsenti porismate, quomodo basis *τετραγωνιζόμενος* Dinostrati beneficio, cuiusvis circuli Quadranti dato recta æqualis describi possit, & cuivis rectæ datæ æqualis peripheria Quadrans. Primo enim, si recta sit ducenda æqualis dato peripheria quadranti: oportet per præsens Theorema, à radio Quadrantis dati auferre basin *τετραγωνιζόμενος*, deinde basi *τετραγωνιζόμενος* & radio invenire tertiam proportionalem, ea enim per præsens porisma æqualis est dato peripheria Quadranti.

Exempli gratia, in præcedente schemate, D N tertia proportionalis basi *τετραγωνιζόμενος* A L & radio A D, æqualis est peripheria Quadranti B C D.

Contra si quadrans peripheria sit decircundans æqualis rectæ datæ; oportet describere quemcunque circuli Quadrantem, rectamque invenire, per Theorema præsens, peripheria Quadrantis descripti æqualem. Quâ obtentâ, reperienda est quarta proportionalis, rectæ inventæ, radio Quadrantis descripti, & rectæ datæ; ea enim est radius Quadrantis circuli postulati. In exemplo, si in eodem schemate detur recta L M, cui æqualem circuli Quadrantis describere oporteat; ubi quadrans A B C D, *τετραγωνιζόμενus* descriptus fuerit, rectaque D N reperta peripheria B C D æqualis, invenienda est A L quarta proportionalis rectæ D N, radio A D, & datæ rectæ L M; ea enim



est radius circuli Quadrantis A P L, ejusque peripheria P L est æqualis datæ rectæ L M. Quæ fuere præstanda.

7. Si tertia proportionalis dicta fiat circuli radius, radius Quadrantis dati erit basis *τετραγωνίου*.

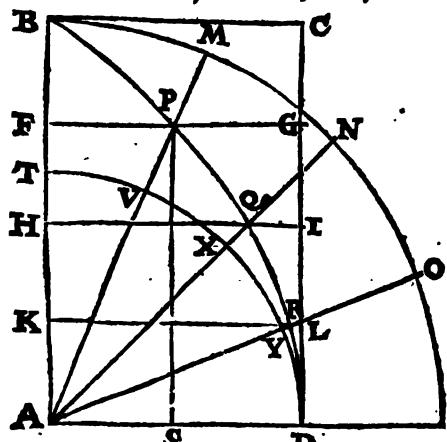
Manente superiore Diaphaga, sit datus circuli quadrans A P L, & M L tertia proportionalis basi *τετραγωνίου* A S & radio R S, id est A L: fiat autem M L id est A D radius Quadrantis circuli B C D; Dico A L radium circuli quadrantis A P L esse basin *τετραγωνίου*. Est enim per 4th Sexti Euclidis.

Ut A L ad M L, ita A D ad D N.

Quamobrem cum D N sit tertia proportionalis radio A D, & A L manifestum est ex Theoremate præmisso A L esse basin *τετραγωνίου*. Quod erat ostendendum.

8. Si rectangulum dati circuli quadrantis radio & tertia proportionali dicta concentum describatur, ejusq; latera majora in quovis partes aequales dividantur, quadransque circuli tertie proportionalis radio descriptus in partes aequales totidem; deinde per puncta divisionum laterum majorum parallela ducantur, radiiq; in puncta sectionum quadrantis: ubi horum singuli secant illarum singulas, puncta sunt linea *τετραγωνίου* Dinostrati, lineaq; uniformiter per ea in terminum basis *τετραγωνίου* ducta, est ipsa linea Dinostrati optata.

Admiranda est natura lineæ *τετραγωνίου*, quia non modò per eam circulus quadratur, & peripheria circuli in rectam lineam extenditur, sed & multa alia perficiuntur quæ magnum usum habent in Geometria, & scitu perjucunda sunt. Rejecta quidem est ea ipsa linea à Pappo Alexandrino, & Sporo Niceno tanquam inutilis, sed non aliam ob causam; quia in quod eos via lateret ipsius terminum deprehendendi: quo latente ipsa linea revera est inutilis. Verum quia & Theoremate via nobis munita est, terminum *τετραγωνίου* obtinendi, non potest non expedita esse ipsius lineæ descriptio, sicuti præmissum Theorema docet, cuius *enunciatio* subjicio.



Describatur rectangulum A B C D contentum dati circuli quadrantis radio A D, & tertia proportionali A B, lateraq; A B & C D majora dividantur in quatuor partes aequales; quadransque A B E, descriptus radio tertie proportionalis A B in partes aequales totidem: deinde per puncta divisionum laterum majorum, ducantur parallela F G, H I, K L, radiique A M, A G, A O, per puncta sectionum quadrantis. Ubi autem radius A B secat parallelam B C, & radius A M parallelam F G, item radius A N parallelam H I, denique radius A O parallelam K L, nimirum in signis B, P, Q, R, sunt puncta lineæ *τετραγωνίου* Dinostrati, lineaque B P Q R D

per ea uniformiter ducta in terminum basis *τετραγωνίου* D, est ipsa linea Dinostrati optata. Nam radius A B circa centrum A per peripheriam B M N O E eodem tempore movetur aequali motu, quo latus B C itidem aequali motu fertur deorsum per latera A B & C D, idque prorsus ut Dinostratus imaginatus est. Hinc fit, ut quartido radius A B pertransivit quamcunque partem arcus B M N O E, tunc latus B C aequales partes laterum A B, D C percurrerit. Habet enim & hic locum prima Archimedis propositio in Helicibus, Si punctum lineæ aequavelociter permeaverit, spacia permeata erunt aequalia temporibus. Unde etiam manifestum est lineam *τετραγωνίου* Dinostrati esse ex familia Helicum, ut rectè judicavit incomparabilis vir Iosephus Scaliger. Enimvero ordinata Helix Cononis aut Archimedis describitur à punto quod aequavelociter percurrit circuli radium & peripheriam;

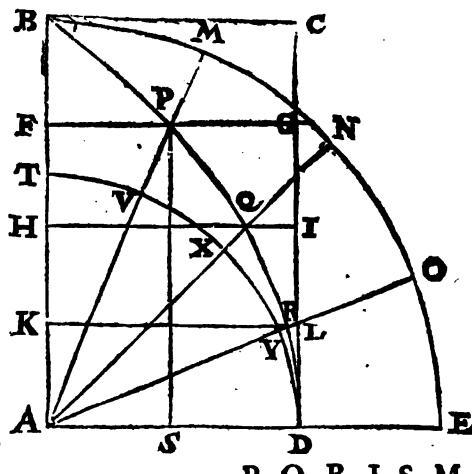
ita etiam *circumferentia* describitur à punto quod aequali veloci motu permeat latera A B, D C, & peripheriam Quadrantis B M N O E. Discrimen tamen inter utramque manifestum est. Ordinata enim Helix aequalibus radii decrementis describitur, at *circumferentia* decrementis inaequalibus, nimirum A B, A P, A Q, A R, A D, ut non sine causa idem Scaliger existimarit *circumferentia* esse illius *rectilineorum*, hoc est, volutam luxatam aut defumbatam. Non sentimus autem cum ipso descriptione *circumferentia*, solo quadrantis intervalllo perficiendam esse: *circumferentia* enim peculiarem circinum vindicat, cuius loco si Mechanicus circuli circino velit uti, præstabat inaequalia intervalla A B, A P, A Q, A R sumere, quam-unicum intervallum A B; nisi malit per tria quavis puncta arcus ducere, atque ita *circumferentia* completere.

P O R I S M A.

*Licet igitur circuli cuiusvis dati quadranti *circumferentia* adscribere.*

Nam per 6 elementum datur tertia proportionalis basi *circumferentia* & radio Quadrantis dati: per præfens aditem elementum describitur rectangulum tertia proportionali & radio quadrantis dati contentum, item circuli quadrans tertie proportionalis intervalllo: atque hinc tandem *circumferentia* ipsa.

9. Si quadranti dato adscripta fit *circumferentia* perpendicularis à quocunque *circumferentia* punto in basin, aequalis est arcui quem recta ex centro quadrantis ducta in *circumferentia* punctū, abscondit ex dicto peripherie quadrante.



P O R I S M A P R I M U M.

Hinc primò reperire possumus circuli cuiusvis arcui dato rectam aequalem, si modò Quadranti circuli dati tetragonizousa adscripta fuerit. Nam si datus arcus sit circuli quadrans, tertia proportionalis est aequalis arcui dato. At si arcus datus quadrante sit minor, ubi per terminum dati arcus in tetragonizousa recta coniuncta fuerit, perpendicularis à punto sectionis tetragonizousa in basin, est arcui dato aequalis. Si vero datus arcus major fuerit circuli quadrante, reperienda primùm est recta aequalis quadranti, vel semicirculo, vel tribus quadrantibus; deinde alia recta aequalis reliquo arcui qui minor est quadrante. Nam due ha recte conjunctæ sunt toti arcui dato aequales.

Iteretur & hic præcedens figura; estoque circuli dati quadrans T D, & tetragonizousa eidem adscripta B P Q R D. Jam si arcus datus sit quadrans D T, aequalis ei recta erit A B, est enim perpendicularis à termino tetragonizousa B in basin A D. Quod si detur arcus V D, erit

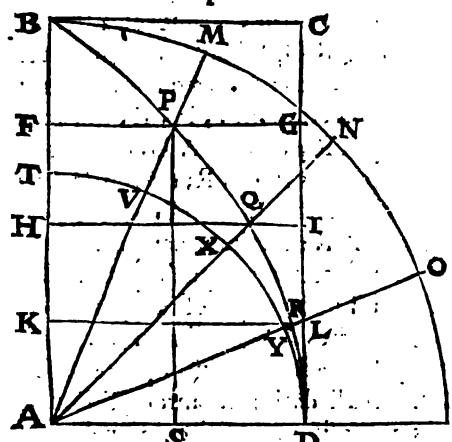
Repetatur superius diagramma; sitq; A T D datus circuli quadrans, & B P Q R D tetragonizousa eidem adscripta: ducatur quoque recta A P ex A centro Quadrantis in P punctū tetragonizous. Dico perpendicularē P S à punto tetragonizous P, in A D basin, aequalem esse arcui D V, quem recta A P abscondit in quadrante D T. Quoniam enim P S id est F A, talis pars est ipsius A B ex constructione, qualis E M est ipsius E B, vel D V ipsius D T; A B autem per confectorum 6 elementi, aequalis est quadranti D T; est etiam per 11 Quinti Euclidis P S aequalis arcui D V. Quod erat demonstrandum.

erit ei æqualis perpendicularis PS demissa ex P punto *tertia proportionalis* in basin AD: recta enim AP ex A centro Quadrantis duxta per V terminum arcus dati secat *tertia proportionalis* in P. Postremo si detur quadrans TD & arcus VD simul, erit ei æqualis recta AB & PS simul. At si duo vel tres Quadrantes dentur in proposito circulo, unâ cum arcu VD, erit ei æqualis recta AD vel bis vel ter sumpta, cum recta PS semel. Quæ erant præstanta.

P O R I S M A S E C U N D U M.

Secundò cuilibet rectæ data abscindere possumus æqualem arcum ex dato quovis circulo, cujus circumferentia non est minor quam recta data; si prius dati circuli quadranti *tertia proportionalis* fuerit adscripta. Nam si recta data æqualis fuerit *tertia proportionalis*, quadrans circuli dati æqualis est rectæ data. Quid si minor fuerit recta data quam *tertia proportionalis*, oportet ex ea abscindere rectam datam & à termino abscissa parallelam basi *tertia proportionalis* ducere in *tertia proportionalis*, rectamq; ex centro quadrantis in punctum concursus parallela & *tertia proportionalis*, arcus enim quadrantis quem eadem recta abscindit, est data rectæ æqualis. Tandem si recta data major fuerit quam *tertia proportionalis*, abscindere oportet hanc ex illa quoties licet, vel semel, vel bis, vel ter, reperiendusq; est arcus reliquo æqualis, ut supra. Hic autem vel uni, vel duobus, vel tribus quadrantibus conjunctus, prout vel semel, vel bis, vel ter, *tertia proportionalis* ex data recta abscissa fuit, componit arcum datae rectæ æqualem.

In eodem schemate, Esto dati circuli quadrans TD, eiique adscripta *tertia proportionalis* BP QRD: sicque primum recta data æqualis *tertia proportionalis* AB, et ei æqualis quadrans TD, ut supra demonstratum est.



Quod si data recta sit æqualis ipsi AF, adeoque minor *tertia proportionalis* AB; oportet eam ex AB abscindere & ex ejus termino F basi *tertia proportionalis* AD parallelam FP ducere in *tertia proportionalis*, & à punto P concursus parallela & *tertia proportionalis* perpendicularis PS demittere in basin; deinde ex A centro rectam ducere in P, quæ per præsens elementum abscindet ex TD Quadrante arcum VD æqualem rectæ datae. Postremò, si data recta sit æqualis *tertia proportionalis* AB & rectæ AF simul, oportet hanc abscindere ex AB, & invenire arcum æqualem reliquo, nimirum arcum VD: hic enim conjunctus cum quadrante TD, æqualis erit rectæ data.

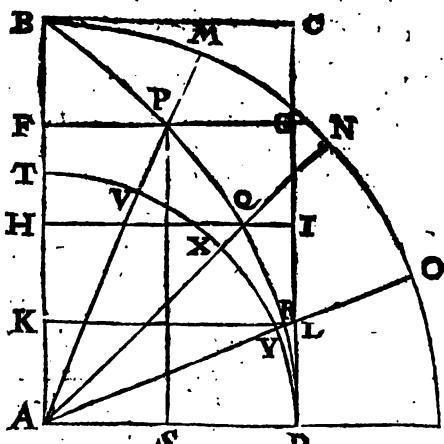
Quod si recta data vel bis, vel ter metiatur AB, & rectam AF semel, oportet Quadrantem TD vel bis, vel ter conjugere arcui VD, prout recta data vel bis vel ter metitur AB, cumq; addere arcui VD, eruntq; hi arcus conjuncti rectæ datae æquales. Quæ facienda erant.

P O R I S M A T E R T I U M.

Tertio datum circuli arcum dividere licet in proportionem datam, si dati circuli quadranti *tertia proportionalis* fuerit adscripta. Nam si arcus datus fuerit arculi quadrantis oportet tertiam proportionalem dividere in proportionem datam, & ex puncto divisionis parallelam basi tetragonizouſis ducere in tetragonizouſan, rectamq; ex centro quadrantis in punctum concursus parallela & tetragonizouſis; hec enim fecabit circuli quadrantem in proportionem datam.

Quod si datus arcus duobus quadrantibus, vel tribus, vel quatuor aequalis fuerit, secare oportet unum quadrantem in proportionem datam, & duplos arcuum sectorum sumere, si datus arcus duos quadrantes aequaliter; vel triplos, si tres; vel quadruplos, si quatuor. Dupli enim dividunt duos quadrantes, tripli tres, quadrupli quatuor, ut simplus unum.

At si arcus datus minor fit quadrante, oportet rectam ducere in terminum arcus dati, & ex puncto sectionis perpendiculariter in basin, & aequalem ei abscindere ex tertia proportionali: Hec deinde in proportionem datam secunda est, & ex punctis sectionum parallelarum basi ducenda in rectaque ex centro quadrantis per puncta intersectionum parallelarum & sectionum; haec enim arcum datum dividunt in proportionem datam. Si vero datus arcus sit quadrante major, secetur primum quadrans, deinde reliquus arcus in proportionem datam, & secuti arcus conjungantur; ita datus arcus in datam proportionem secutus erit. Denique si datus arcus sit quadrantibus duobus vel tribus major; secandi primum sunt vel duo quadrantes, vel tres in proportionem datam, deinde arcus reliquus; nam & hi conjuncti propositum arcum in datam proportionem dividunt.



Datus esto in eadem diaphaga quadrans $T D$, quem dividere oporteat in proportionem datam, puta quadruplam, siveque ei adscripta $B P Q R D$. Dividatur primum tertia proportionalis AB in partes aequales quatuor, deinde ex punctis sectionum F, H, K , ducantur rectae FP, HQ, KR parallelae AD , quae secent AO in punctis P, Q, R ; tandemque ex A centro agantur in puncta sectionum V, X, Y rectae AP, AQ, AR ; secabunt haec quadrantem TD datum in proportionem quadruplam. Nam AB ex praesenti Theoremate est aequalis Quadranti TD , & partes AB partibus TD . Cum igitur partes AB quadrantes sint, oportet etiam partes TD quadrantes esse; adeoque rectam AB , & circuli dati quadrantem TD divisi sunt in proportionem quadruplam. Quod si arcus propositus, semicirculo, vel tribus Quadrantibus, vel etiam semicirculo aequalis fuerit, oportet nihilominus Quadrantem ut supra secare in proportionem quadruplam, sed si semicirculum eodem modo dividere libeat, sumendus est duplus ipsius DY , viz. DX , hic enim est semicirculi quadrans. Aut si peripheria tribus quadrantibus constans eodem modo secunda sit, triplus arcus, nimirum DV est capiendus, Nam & hic tres Quadrantes circuli una peripheria contentos dispescit in proportionem quadruplam. Tandem si circulus eodem modo dividendus sit, sumendus est quadruplus arcus DT : hic enim quia est dati circuli quadrans, utique eundem circulum secat in proportionem eandem. At si detur arcus VD , minor Quadrante circuli TD , siveque dividendus sit in proportionem triplam; oportet rectam ducere ex A centro quadrantis per V terminum arcus dati, que secabit in punto P , ex quo demittenda est perpendicularis PS in basin AD , & ex AB abscindenda est AF aequalis PS . Haec deinde dividenda est in proportionem triplam, & ex punctis sectionum F, H, K , ducenda sunt FP, HQ, KR , parallelae basi AD , quae secabunt in punctis P, Q, R .

Cyclometriae Liber I.

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Q, R. Tandem ex A centro quadrantis rectæ agendæ sunt in puncta Q & R, quæ divident arcum V D in proportionem triplam. Nam arcus V D, ex præsenti Theoremate est æqualis rectæ A F, & illius partes F H, H K, K A, sunt æquales arcibus V X, X Y, Y D. Quare cùm partes A F sint trientes, oportet & partes arcus V D esse trientes, rectamque A F, & arcum V D, divisum esse in proportionem triplam. Tandem si arcus datus quadrante sit major, oportet primum quadrantem, deinde reliquum arcum secare in proportionem datam, & partes quadrantis singulas, singulis partibus arcus reliqui addere, sic enim dividetur arcus propositus in proportionem datam. Atque ita etiam est procedendum, cum peripheria datur duobus, vel tribus quadrantibus major; nisi quod partes quadrantis secuti in proportionem datam vel bis, vel ter sumendæ sint, prout peripheria data vel duobus vel tribus quadrantibus est major.

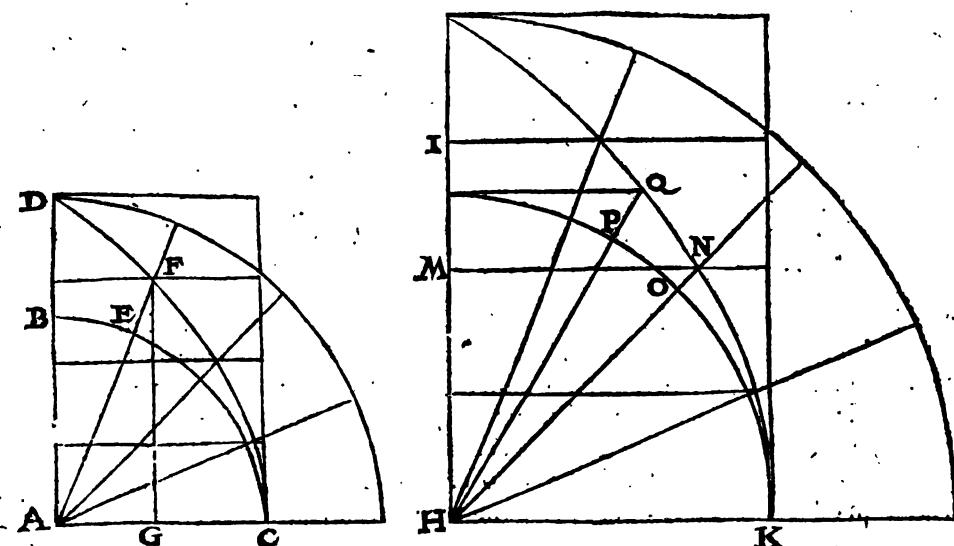
Est autem præsentis porismatis in Geometria magnus usus. Primum enim ipsius admissiculo quæcunque figuræ, sive parium sive imparium laterum, circulo dato inscribuntur; circulusque ipse, & quævis ejus peripheria data, in datam proportionem dividitur.

Secundò quivis angulus datus potest dispefi in proportionem datam. Quia enim per 33. Sexti Euclidis arcus ad arcum est, ut angulus ad angulum, haud dubiè quod de arcibus est demonstratum, de angulis una opera demonstratum esse oportet.

Tertiò Triangulūm Isosceles construi potest, cujus uterque æqualium angulorum ad reliquum habeat proportionem datam: Unde etiam artificium pendet quamcunque figuram circulo adscribendi: quod tamen, ut Proclus censet, difficile est rudibus, quia multiplex & varium est opus.

P O R I S M A Q U A R T U M.

Postremò propositis duobus inæqualibus circulis, datoque arcu in alterutro, possumus æqualem abscindere ex altero; si modò utiusque circuli quadranti^e circuari^e sit adscripta. Oportet autem arcum in majore circulo datum, non esse minore circulo dato majorem. Nam si arcui dato inveniatur æqualis recta per primum porisma; & hinc recta æqualis arcus in altero circulo, erit hic arcus inventus arcui dato æqualis.



Sint in præmissis figuris, A B C quadrans circuli minoris, & H I K majoris, quibus sigillatim adscripta sit circuari^e D C & L K. Deturque primum in minore circulo arcus E C, cui æqualis abscindendus ex majore circulo. Per primum porisma hujus elementi,

perpendicularis FG est æqualis arcui dato EC . Per secundum verò porisma hujus, rectæ FG , cui æqualis est ipsa HM , est etiam æqualis arcus OK . Itaque per ii Quinti Euclidis, arcus OK majoris circuli, & EC minoris sunt æquales. Abscissus igitur est ex maiore circulo arcus OK , dato EC in minore circulo æqualis. Quod faciendum erat.

Secundò detur in majore circulo arcus PK , cui æqualis abscindendus sit ex minore. Primum perpendicularis HI æqualis est, arcui PK per primum porisma hujus. At per secundum porisma rectæ AD (quæ facta est æqualis ipsi HI) est etiam æqualis arcus BC . Ergo per ii Quinti Euclidis, Quadrans BC , & arcus PK sunt inter se æquales. Abscissus itaque est ex minore circulo arcus BC , æqualis PK dato in circulo majore. Quod facere oportebat.

Atque ita pertractata est prima Cyclometriæ pars, de dimensione circuli ambitus: sequitur altera de dimensione circuli areæ, que sequenti libro est explicanda.

C Y C L O M E T R I Æ L I B E R II.

De dimensione Circuli areæ.

1. Altera pars Cyclometrie est que benè metitur circuli aream.

Anc partem Cyclometriæ Græci τετραγωνίου κύκλου, nostri Quadraturam circuli appellant. Est autem nobile argumentum quod omnium æratum Mathematicis, propositum fuit, ut in eo se exercent: pendentque à ratione diametri & peripheriæ; adeò ut ea inventa τετραγωνίου κύκλου sponte sequatur. Itaque dubium non est, quin pars isthac Cyclometriæ perfacilis jam sit ferta, quia diametri & peripheriæ ratio, superiore libro satis superq; est demonstrata.

2. Aream circuli dimetiri, est non tantum circulo cùcunque dato æquale quadratum describere, & curvis quadrato dato æqualem circulum, sed & rationem explicare quam circulus quisque datus habet ad quadratum sui diametri.

Areæ circuli diimensio vel Geometricè fit, vel Arithmeticè. Si Geometricè, describere oportet quadratum circulo dato æquale, vel circulum æqualem dato quadrato. Sin Arithmeticè, explicanda est ratio quam circulus habet ad quadratum sui diametri.

3. Rectangulum cujusvis circuli radio, & peripheriæ dimidio contentum, æquale est eidem circulo.

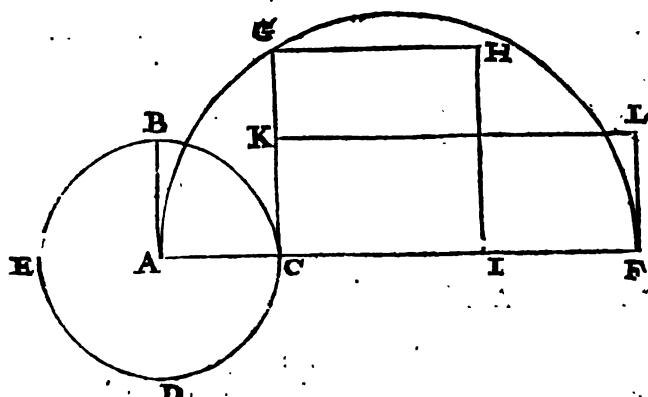
Archimedes prima propositione μετόποις κύκλου demonstrat omnem circulum esse æqualem Triangulo rectangulo cujus unum latus circa rectum est circuli radius, alterum perimeter. Demonstratio autem fumitur à dimensione areæ cujusvis polygoni ordinati, quæ à minimo ad maximum uniformiter se habet. Triangulum enim rectangulum cujus unum latus rectum ambiens est perpendicularis à centro polygoni in latus, & alterum perimeter polygoni, polygono æquale est. Cùm verò circulus sit polygonum ordinatum infinitorum laterum, dubium non est, quin polygonum ordinatum finitorum laterum ad circulum quoque se extendat, quia eadem utrobique est ratio. Adeoque verissimum est quod Archimedes asserit, omnem circulum esse æqualem Triangulo rectangulo, cujus unum latus est circuli radius, alterum ipsius perimeter.

Hinc autem varia extiterunt Axiomata apud Theonem & alios, & inter cætera illud quod nos adduximus, rectangulum circuli cujusvis radio & peripheriæ dimidio contentum, æquale esse

esse eidem circulo. Cujus veritas cum ex Archimedeo elemento, cognita parallelogrammi doctrina, sit manifesta, non opus est, ut pluribus ostendatur.

P O R I S M A P R I M U M.

Primo itaq; cuivis circulo dato licet describere aequalē quadratum. Media enim proportionalis inter radium circuli, & semissem perimetri est latus quadrati, circulo dato aequalis.



Detur in adjuncto schemate circulus BCDE, ejusque radius AC, & semiperimeter CF per 1 porisma 5 Cyclometria, vel porisma 6 elemen. sitq; circulo dato describendum aequalē quadratum; Dico medium proportionale inter radium AC, & semiperimetrum CF esse latus Quadrati dato circulo aequalis. Quadratum enim quod describitur à media proportionali, inter radium circuli AC, & perimetri semissem CF, aequalē

est (per 17 Sexti Euclidis) rectangulū quod continetur radio AB id est KC & perimetri semissem CF. Quoniam vero per præsens Theorema, hoc ipsum rectangulum, circulo BCD E aequalē est; utique & Quadratum descriptum à linea media, eidem circulo est aequalē. Inventā igitur, per 13 Sexti Euclidis CG media proportionali inter radium AC, & semipermetrum CF, datur quadratum CGHI, circulo BCD E aequalē. Quod erat faciendum.

Manifestum vero est ex demonstratione præmissa, figuram quoque quamcunque rectilineam cuivis circulo dato posse construi. Nam si per præsens porisma dato circulo aequalē quadratum construamus, & per 25 Sexti Euclidis, eidem quadrato figuram rectilineam aequalē, & similem alteri datæ rectilineæ figuræ; erit eadem figura rectilinea constructa dato circulo aequalis.

P O R I S M A S E C U N D U M.

Secundō cuicunque quadrato dato describi potest aequalis circulus, si prius per præcedens porisma, cuivis circulo aequalē quadratum descriptum sit. Quarta enim proportionalis descripti, dative quadrati lateri, & radio dati circuli, est radius circuli, dato quadrato aequalis.

Detur in adjuncto schemate quadratum ABCD, cui aequalis circulus describendus sit, sitque circulo AKL aequaliter descripto, aequalē quadratum, per præcedens porisma AFMN. Dico quartam proportionalem lateribus AF & AB, & radio AE, esse radius circuli, dato quadrato ABCD aequalis. Describatur enim recta linea EF ex E punto in punctum F, & ex B punto ducatur BG parallela ipsi EF, eruntque Triangula EAF & GAB similia, & latera Triangulorum, per 4 Sexti, proportionalia. Itaque

Ut AF ad AB, ita AE ad AG quartam proportionalem, lateribus quadratorum AF, AB, & radii AE.

Decircinetur quoque radio AG circulus AHI, eritque per demonstrata Pappi,

ut

